The Golden Ratio (part I)

Lesson Plan

Cube Fellow: Matthew Wells **Teacher Mentor: Kimberly Halsey**

Goal: **This lesson is designed to be a two part lesson. The first part focuses on the idea of ratio and proportion via looking at areas of rectangles. The main goal of this lesson is to provide a conceptual approach to understanding a proportion. The students will then be able to apply this approach to verifying whether two ratios are proportional or not. In the end, the ideas presented will be directly useful when part II is implemented. Throughout this description, the focus will be on the first part of the lesson.**

Grade and Course: **Ninth-Tenth grade Algebra I course**

KY Standards: **MA-HS-5.2.2 : Students will evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified values of their variables.**

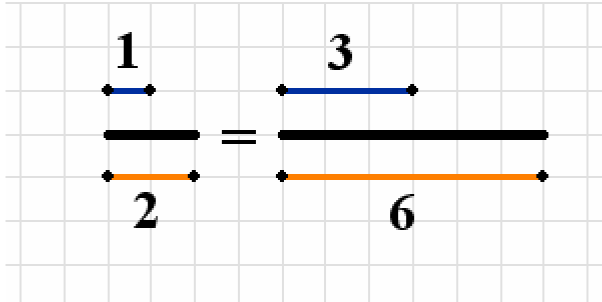
Objectives: **The students will be able to:**

1) **Verify when two ratios are proportionate.**
2) **Solve for an unknown given a proportion.**
3) **Set up and solve ratios and proportions.**
4) **Understand how to verify a proportion is true using an area model.**

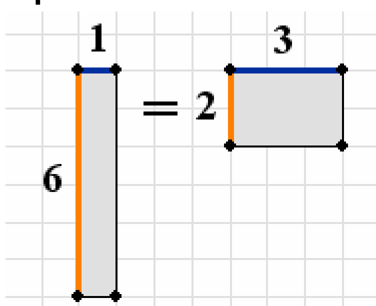
Resources/materials needed: **Pencil, paper (graph paper optional)**

Description of Plan: **Coming into this lesson, the expectation is that the students have been introduced to ratios (and proportions) in a previous class. For part I of this lesson, the instructor will be lecturing the class. Although this may seem boring and uneventful, the ideas discussed should greatly enhance the second aspect of this lesson. The lesson begins with a general reminder about what a ratio and proportion are. Reminding them through examples should get them in the correct mindset. On the chalkboard (or marker board), a thorough definition of ratio and proportion should be given.**

Next, the instructor should provide an example of a simple true proportion, such as 1/2 = 3/6, which the students know to be true. The focus should then shift to how we can conceptualize this proportion. A picture of this proportion should be drawn on the board (and on their papers) as follows:

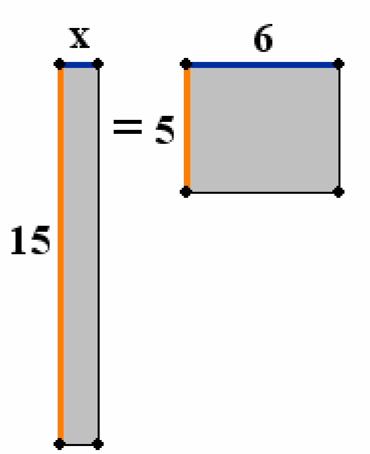


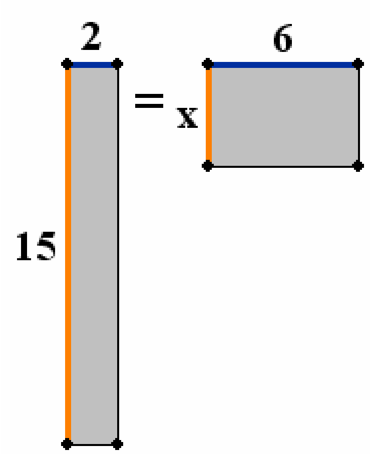
From this picture, it is easy to see that when the blue segment gets three times longer, the orange segment similarly gets three times longer. Now we can use this picture to aide in the understanding of cross multiplication. We find that 1 x 6 =
2 x 3; this is given by the picture below:



Here, we are re-enforcing the concept that multiplication of two numbers can be thought of as the area of a rectangle. The picture also clearly demonstrates that the area of the two objects is 6, and thus verifying that the proportion was indeed true. Color coding the proportion is also useful, as we want to distinguish the numerator and denominator of the ratio. (Note: This will help the students avoid setting up proportions where they have inverted the ratio; i.e. mph = hpm)

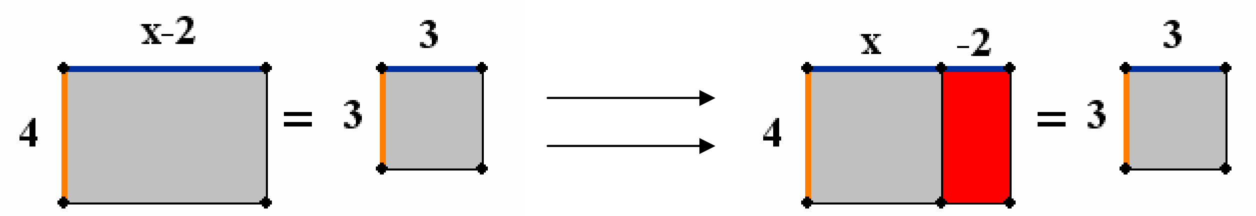
After a concrete example, the goal should be to talk about proportions involving an unknown quantity (a.k.a. variable). The above ideas involving areas of rectangles can still be used. The instructor should give an example where the proportion is known to be true, but there is a piece missing (e.g. *x* / 5 = 6 / 15 or
2 / *x* = 6 / 15). The same pictures can be drawn. Below are the rectangular regions of the two proportions given:





Now ask the students what the areas of the rectangle are. The students should be familiar with length x width, and so they will say 15x, 30, 30, and 6x for the areas above. The setup now allows the instructor to note that we want the areas of rectangles in proportions to be equal, and so 15x = 30 and 30 = 6x. The students can now use their algebra skills to solve these equations.

In the final stage of the lesson, the instructor will go over how to think about proportions involving linear terms and negative values. Again, with a little forethought, we can use the same model. For example, (x – 2)/ 3 = 3 / 4 can be represented as:



The first picture can be interpreted as the previous pictures where (in terms of area). Of course, we end up solving 4(x-2)=9. The second picture re-enforces the idea of distribution and also distinguishes negative "areas" with positive "areas". There should be something said about how we get a red region when a negative is multiplied to a positive. Certainly the red region represents 4 x (-2) = -8. The grey regions are 4x and 9 respectively. Thus we ultimately want the areas equal, so
4x-8 = 9, which is 4(x-2)=9 distributed. Note that modeling proportions in this manner will have an added bonus of a different interpretation of the distribution property. Lesson one is now finished, of which we will uses these ideas in the second part of the lesson.

Lesson Source: Mathematical Reasoning for Elementary Teachers by Long and DeTemple (the discussion of the area model for multiplication sparked my imagination for lesson I of this project)

Instructional Mode: Lecture and note-taking

Date Given: December 12, 2006 **Estimated Time:** One 45-minute class period

Date Submitted to Algebra^3: July 2, 2007

Form 8-18-06

The Golden Ratio (part II)

Lesson Plan

Cube Fellow: Matthew Wells **Teacher Mentor**: Kimberly Halsey

Goal: This is the second part of the Golden Ratio lesson. The first part focused on the idea of ratio and proportion via looking at areas of rectangles. With this understanding in mind, the students will then be asked to investigate ratios that happen in nature, such as height versus wingspan. Emphasis will be put on creating pictures of these ratios as in lesson I. Throughout this description, the focus will be on the second part of the lesson.

Grade and Course: Ninth-Tenth grade Algebra I course

KY Standards: MA-HS-5.2.2 : Students will evaluate polynomial and rational expressions and expressions containing radicals and absolute values at specified values of their variables.

Objectives: The students will be able to:

1) Verify when two ratios are proportionate.
2) Solve for an unknown given a proportion.
3) Set up and solve ratios and proportions.
4) Understand how to verify a proportion is true using an area model.
5) Apply these ideas to ratios that appear in nature

Resources/materials needed: String (different colors preferable) and scissors, enough for four groups. Rulers are optional.

Description of Plan: Since this is the second part of a two-part lesson, be sure that you have discussed some of the ideas from the first lesson. Otherwise, some of the ideas here might seem awkward and strange.

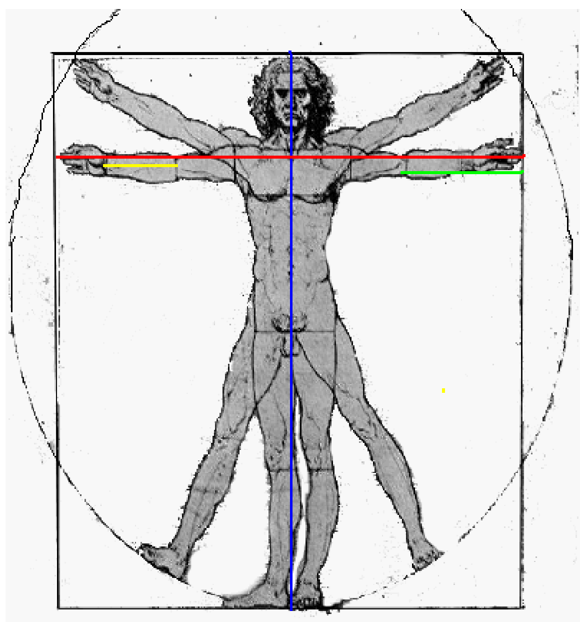
This lesson is focused on measuring different lengths on a person's body using pieces of string. The overall goal is to then use the string to discuss ratios. So, to begin, select 8 people and pair them up. Give one person from each pair a pair of scissors, and give the other person a roll of string. Each group will measure a piece of string to a person's body part. The measurements for this project, illustrated by DaVinci's Vitruvian Man, are:

GROUP 1: Height – measure from top of head to bottom of feet, given by the blue line

GROUP 2: Wingspan – measure from the person's fingertip to opposite fingertip, given in red below

GROUP 3: Elbow to Fingertip – measure from the end of the fingertip to the inside of the elbow on the same arm, given in green below. Here, the inside of the elbow is where the skin fold when you start to bend your elbow; also, this is where they typically take blood from.

GROUP 4: Elbow to Wrist – measure from the elbow (see group 3) to the wrist, given in yellow below. The area to measure to on the wrist is where the skin starts to fold below your palm when you bend your wrist.



Instruct the rest of the class, in an organized manner, to visit each group to be measured. Have the cutters and measurers also do this after everyone else has. After this is done, each student should have four different pieces of string, which are representative of themselves.

Now the class should re-convene and discuss the equivalence of these ratios. The two ratios of interest are:

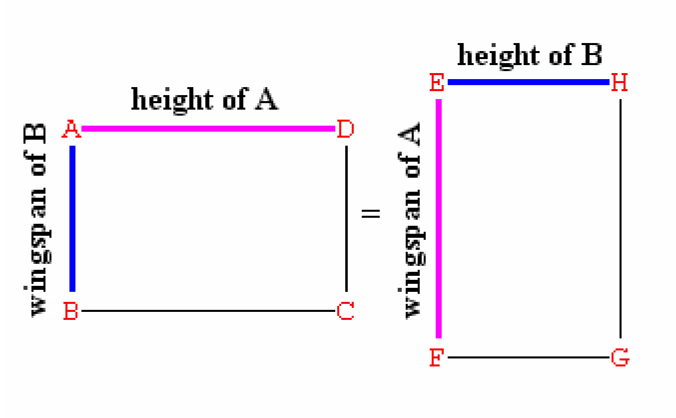
height : wingspan **elbow-fingertip : elbow-wrist**

The ratio of the first is typically 1:1. This ratio should be similar for most people. To get the students to see this, you can use the ideas from the first

lesson. Select the height and wingspan strings from two separate students (preferably one student should be taller than the other). Set up the proportion on the board for all to see:

$$\frac{\text{Height of Student A}}{\text{Wingspan of A}} = \frac{\text{Height of Student B}}{\text{Wingspan of B}}$$

Cross multiply to get (Height A)(Wingspan B) = (Height B)(Wingspan A). Visually, take the strings that represent these different values, and tape them on the board (or lay them on the floor) to create two separate rectangles, as follows:



After some observation, the students should determine that the proportion is correct, as one rectangle is a rotation of 90 degrees of the other rectangle. Apply this idea to the other ratio. Here, the ratio should be about 1.61803 : 1, which is indeed the Golden Ratio! The teacher should challenge the students to verify that the proportion is indeed true by having them compute the areas of the two different rectangles by hand. This works best if the students have rulers and are in pairs.

After the activity, the teacher can choose whether to discuss the Golden Ratio in greater detail. The detail of such discussions is omitted here, but please visit the website listed below for examples of where the Golden Ratio appears in nature.

Lesson Source: The following websites provided me with ideas:
http://www.homeschoolmath.net/teaching/proportions.php
http://goldennumber.net/dna.htm

Instructional Mode: Group activity and Lecture

Date Given: December 13, 2006 **Estimated Time**: One 45-minute class period

Date Submitted to Algebra^3: July 2, 2007

Form 8-18-06