Proving Pythagoras

Lesson Plan

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Goal: Get 10th Graders to understand and prove the Pythagorean Theorem

Grade and Course: tenth grade geometry

KY Standards: Geometry: MA-11-2.1.3 Students will apply definitions and properties of right triangle relationships (right triangle trigonometry and the Pythagorean Theorem) to determine length and angle measures to solve real-world problems.

Objectives: The students will:

1. learn the definition of a right triangle, hypotenuse, and the legs of a right triangle
2. understand that the Pythagorean Theorem is a relationship between areas
3. use the Pythagorean Theorem to find the length of an unknown side of a right triangle
4. prove the Pythagorean Theorem
5. realize that there is nothing mysterious about the theorems from their textbooks, but that they actually have the ability to figure these things out for themselves

Resources/Materials Needed: PowerPoint, an overhead projector, or a chalk board is necessary for the lecture portion of the lesson. Also, each

group will need four identical right triangles made from construction paper (see Fig. 1).

Description of Plan: The lesson begins with a brief lecture introducing right-triangles and the Pythagorean Theorem. The concepts of length and area are discussed, stressing the fact that the Pythagorean Theorem is fundamentally a relationship between areas. It is useful to introduce units when discussing the length of a side of a triangle. This way, when the quantity is squared, it is possible to use the square footage of a house or apartment as an example. The salient features of a right triangle are labeled in class and the lecture concludes with example problems that find the unknown length of a side by using the Pythagorean Theorem. Following the lecture, the class is put into groups and each group is given four identical right triangles, each of which has the hypotenuse and legs labeled. The students are asked to organize their triangles into a square with the four hypotenuses forming the perimeter of the square (see Fig. 1). They are then guided through the proof by being asked a series of questions about the arrangement of triangles.

Lesson Source: The figures used in the lecture slides and the graphical proof were found on the World Wide Web at the following URL:

http://www.cut-the-knot.org/pythagoras/index.shtml

Instructional Mode: The lesson begins with a short lecture introducing definitions, concepts, and working examples. The bulk of the lesson consists of a group activity in which students must work together in a creative manner in order to complete a set of tasks. There is a "hand-on" aspect of the group activity in the form of the construction paper triangles.

Date Given: October 11, 2006.

Estimated Time: It is possible to complete the lesson in a single class

period (around 45 minutes). It would be more comfortable to have the lecture the first day and devote the majority of a second day to the group activity.

Date Submitted to Algebra[3]: November 7th and November 14th, 2006

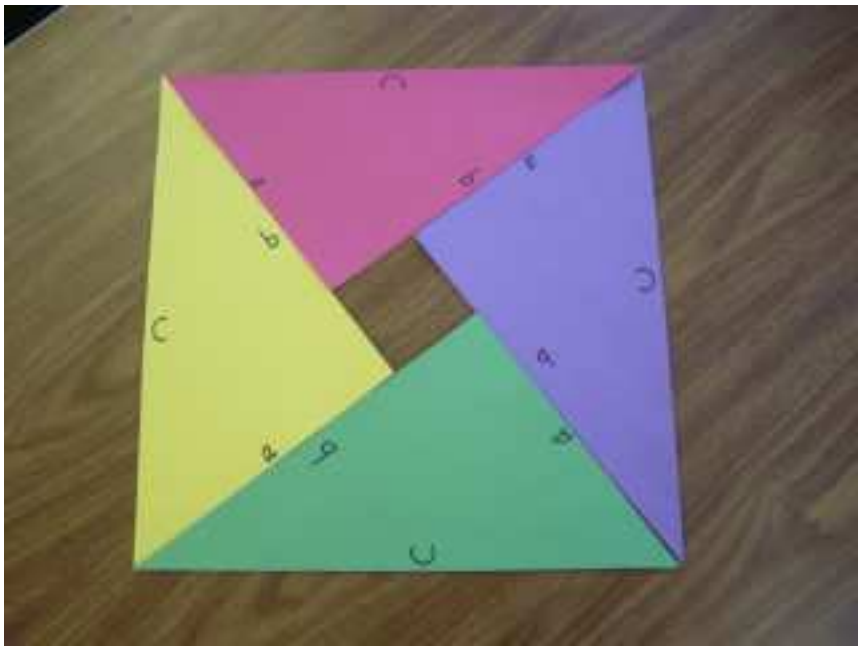
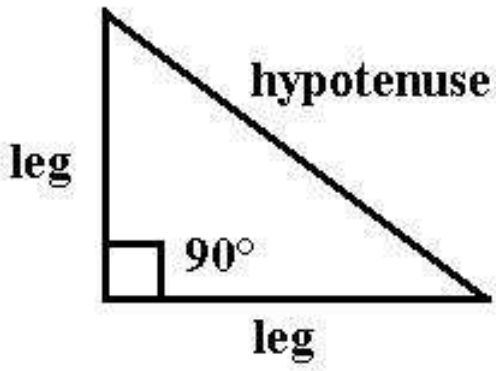


Figure 1: paper triangles for the group activity

LECTURE SLIDES FOLLOWS

RIGHT TRIANGLES AND THE PYTHAGOREAN THEOREM

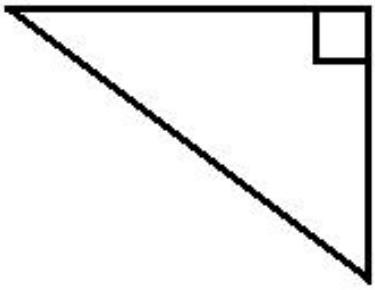


RIGHT TRIANGLE: A right triangle is a triangle in which one of the angles is a 90 deg angle.

LEGS: a right triangle has legs! They are the sides that form the right angle.

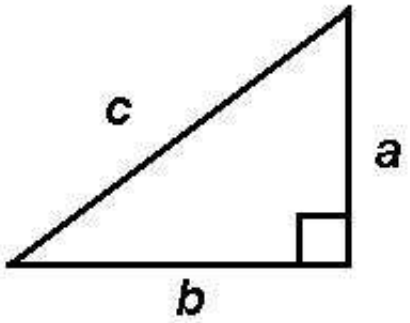
HYPOTENUSE: The hypotenuse is the side opposite the right angle.

Question: Is the following triangle a right triangle? If so, label its legs and hypotenuse.

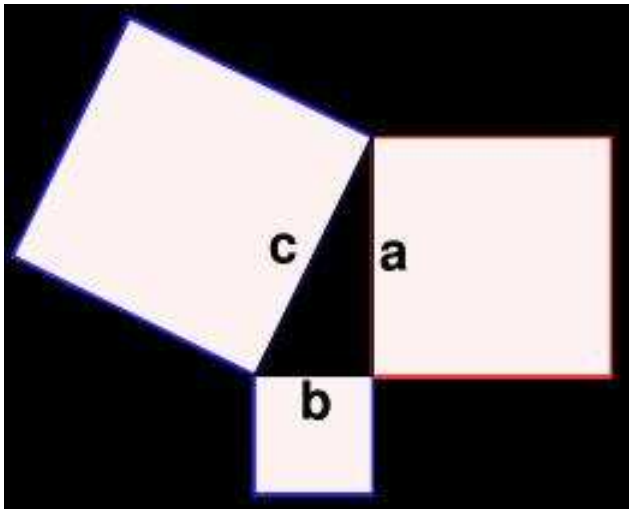


THE PYTHAGOREAN THEOREM: In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

$$c^2 = a^2 + b^2$$

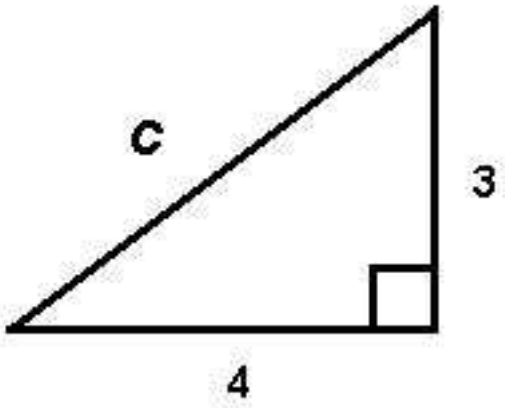


Ok Great! But what does that *mean*?

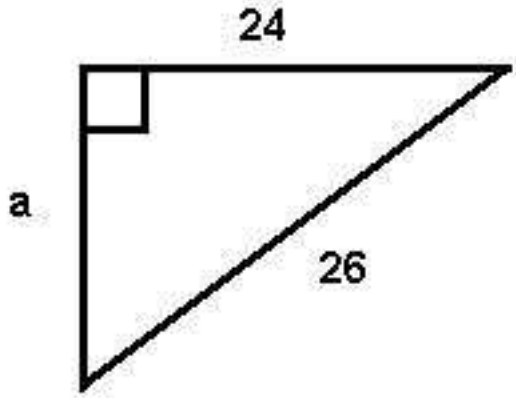


Let's work some problems!

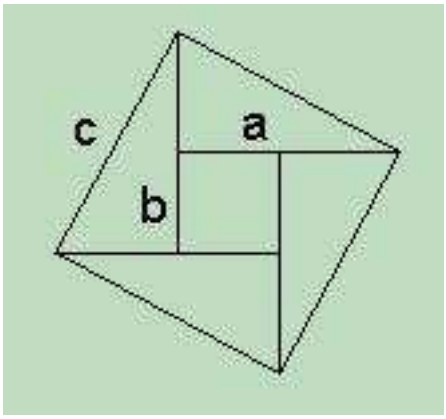
1. Find the length of the hypotenuse.



2. Find the length of side **a**.



PROOF OF THE PYTHAGOREAN THEOREM (in groups)



1. Arrange your triangles to match the drawing above.

2. What is the area of the large square? Write a simple expression that describes the area.

3. What is the area of the small square? Write an expression with a and b that describes the area.

4. What is the total area occupied by all four triangles? Write an expression with a and b that gives the total area of the four triangles. (HINT: what is the area of a single triangle?)

5. What is the relationship between the area of the large square and the combined areas of the small square and the areas of the four triangles? Write an equation describing this relationship. (HINT: $(b-a)^2 = b^2 - 2ab + a^2$)