Average and Instantaneous Rates of Change

1. A train travels from city A to city B, pauses, then travels from city B to city C. The train leaves city A at time 10:00 am and arrives at city B at 12:15 pm. The train leaves city B at 2:00 pm and arrives at city C three hours later. The average velocity of the train, while travelling from A to B, was 45 miles per hour. The distance between city B and city C is 240 miles. What is the average velocity of the train from city A to city C (including the stop)?

2. A train leaves city A at 8:00 am and arrives at city B at 10:00 am. The average velocity of the train from A to B was 60 miles per hour. The train leaves city B at 10:00 am and arrives at city C at 1:00 pm. Find the average velocity of the train from city B to C, given that the average velocity from A to C was 50 miles per hour.

3. Let

$$f(x) = \frac{3}{x^2 + 1}$$

(a) Find the average rate of change of $f(x)$ from $x = 0$ to $x = 2$.

(b) Draw the graph of $y = f(x)$. (A graphing calculator may be useful.) Explain how the rate of change found in part (a) can be represented on this graph.

4. Find a positive number A so that the average rate of change of

$$g(x) = 3x^2 - 1$$

from $x = 2$ to $x = A$ is equal to 33

5. An object is launched up in the air. The height of the object after t seconds is $H(t)$ feet, where $H(t) = -16t^2 + 256t + 64$.

(a) When is the object at its greatest height? (Hint: What must be true about the velocity of the object when it is at the greatest height?)

(b) What is the maximum height of the object?

6. Let $g(x) = x^2 - 4x$.

(a) Find the value of x for which the tangent line to $y = g(x)$ has slope equal to 6.

(b) Find the value of $g(x)$ at the point where the tangent line to $y = g(x)$ is parallel to $y = 2x + 5$.

(c) Find a value of x so that the instantaneous rate of change of g at x is equal to the average rate of change of g from $x = -1$ to $x = 3$.

7. Suppose $q(x) = 3x^2 - 12x + 8$ and $p(x) = 3x^2 - 12x + 5$.

(a) Find $q'(x)$ and $q'(1)$.

(b) Find the equation of the tangent line to $y = q(x)$ at $x = 1$. Write the equation of the tangent line in slope-intercept form.

(c) Find $p'(x)$ and $p'(1)$.

(d) Find the equation of the tangent line to $y = p(x)$ at $x = 1$. Write the equation of the tangent line in slope-intercept form.

(e) Discussion: What do you notice about your answers? Why is this to be expected, given the graphs of $y = p(x)$ and $y = q(x)$?