The v_1-Periodicity Region of the E_2-page of the $\mathbb{C}$-Motivic Adams spectral sequence

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Graduates Reminisce Online On Topology
July 15 2020

E_2-page of the classical Adams spectral sequence

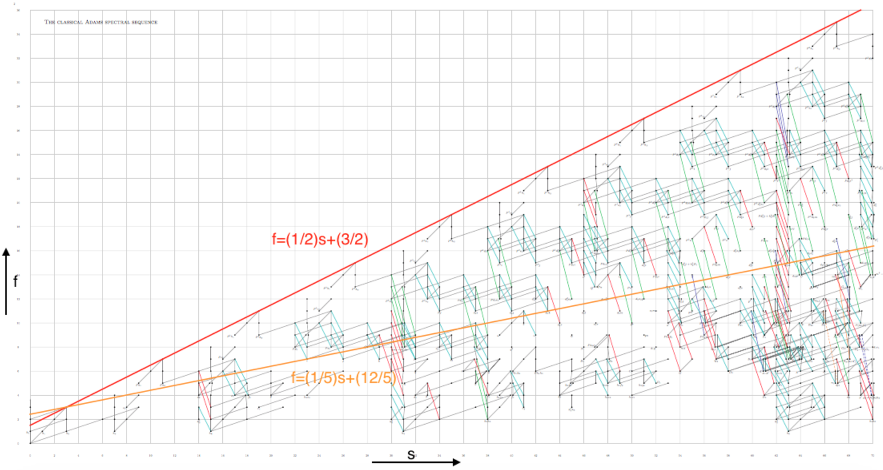

Figure: classical Ext

E_2-page of the classical Adams spectral sequence

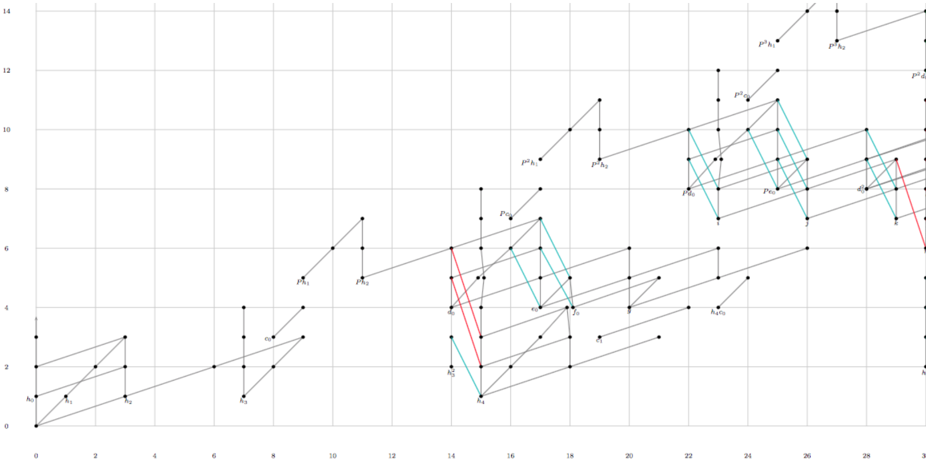

Figure: classical Ext

E_2-page of the $\mathbb{C}$-motivic Adams spectral sequence

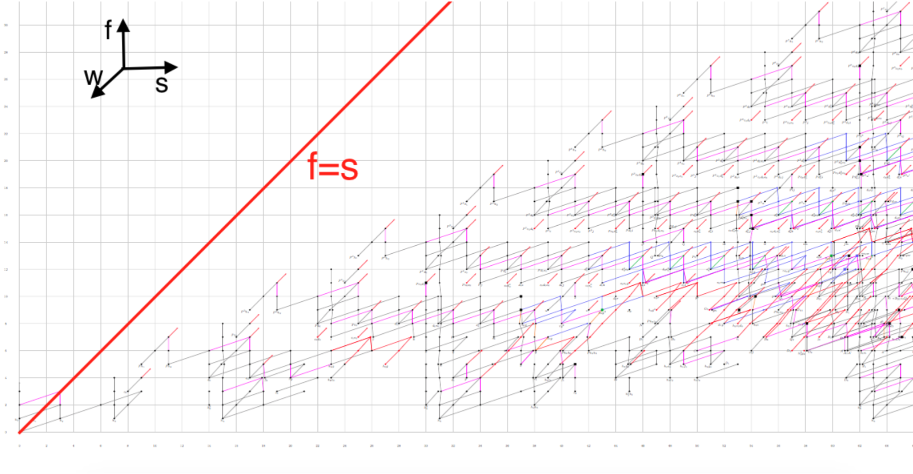

Figure: $\mathbb{C}$-motivic Ext

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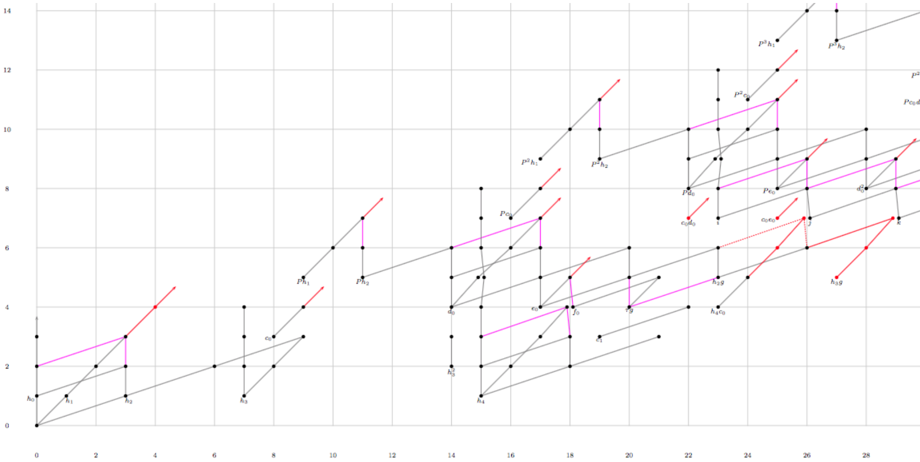

Figure: $\mathbb{C}$-motivic Ext

$[S/h_0, \Sigma^{-1,1,0}F_0]_{*,*,*}^{\mathcal{A}(1)^\vee}$

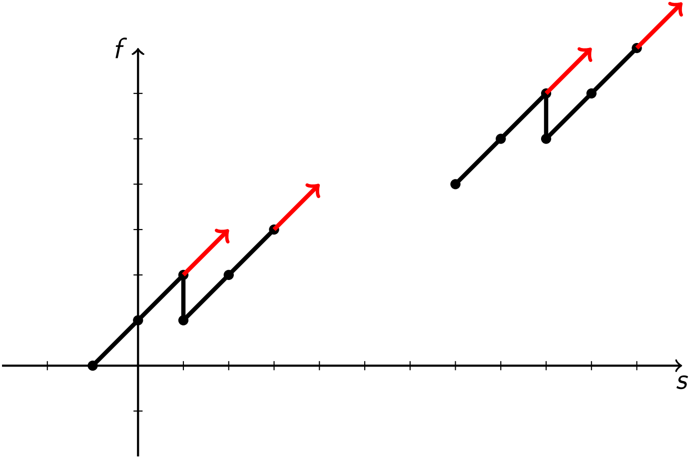

Figure: $[S/h_0, \Sigma^{-1,1,0}F_0]_{*,*,*}^{\mathcal{A}(1)^\vee}$

$[S/h_0, \Sigma^{-1,1,0}F_0/h_1^{\infty}]^{\mathcal{A}(1)^\vee}_{*,*,*}$

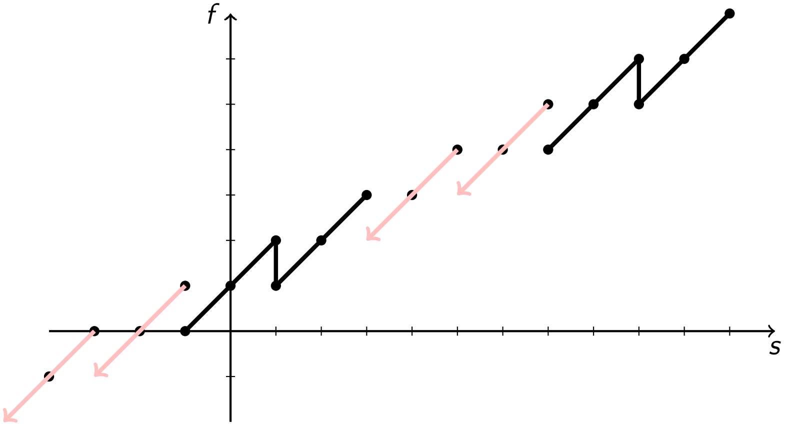

Figure: $[S/h_0, \Sigma^{-1,1,0}F_0/h_1^{\infty}]^{\mathcal{A}(1)^\vee}_{*,*,*}$

$[S/(h_0,\theta), \Sigma^{-1,1,0}F_0/h_1^{\infty}]^{\mathcal{A}(1)^\vee}_{*,*,*}$

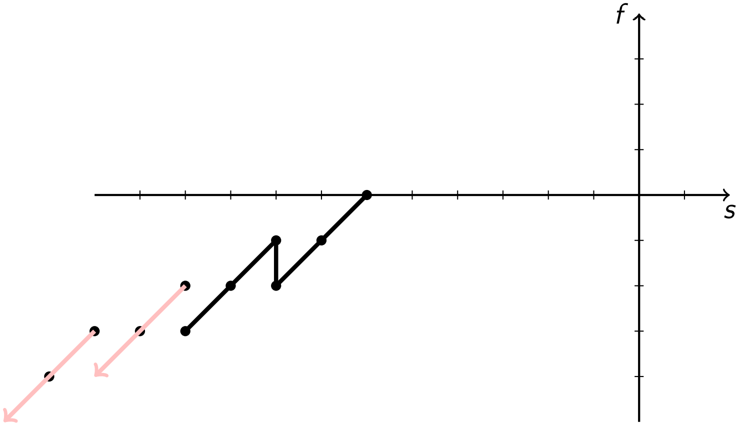

Figure: $[S/(h_0,\theta), \Sigma^{-1,1,0}F_0/h_1^{\infty}]^{\mathcal{A}(1)^\vee}_{*,*,*}$

The Cartan-Eilenberg spectral sequence

Move to $\mathcal{A}(2)^\vee$ by a sequence of normal extensions

$$\mathcal{A}(2)^\vee \to \mathcal{A}(2)^\vee/\xi_1^2 \to \mathcal{A}(2)^\vee/(\xi_1^2, \xi_2) \to \mathcal{A}(2)^\vee/(\xi_1^2, \xi_2, \tau_2) = \mathcal{A}(1)^\vee$$

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First one:

$$E[\tau_2] \to \mathcal{A}(2)^\vee/(\xi_1^2, \xi_2) \to \mathcal{A}(1)^\vee$$

The E_2 page is

$$\mathrm{Ext}_{\mathcal{A}(1)^\vee}(M, N) \otimes \mathrm{Ext}_{E[\tau_2]}(\mathbb{M}_2, \mathbb{M}_2) \cong [M, N][\tau_2]$$

From $\mathcal{A}(1)^\vee$ to $\mathcal{A}^\vee$

Those βs we are throwing in are:

- $\tau_2 = (6, 1, 3)$
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$$[S/(h_0,\theta), F_0/h_1^\infty]^{\mathcal{A}(2)^\vee/\xi_1^2}[\xi_1^2] \Rightarrow [S/(h_0,\theta), F_0/h_1^\infty]^{\mathcal{A}(2)^\vee}$$

From $\mathcal{A}(2)^\vee$ to $\mathcal{A}$, every elements we throw in will have "slope" lower than $\frac{1}{5}$!

The motivic periodicity theorem

$$\begin{array}{ccccccc}
[S/(h_0^k,\theta),F_0/h_1^\infty]_{s,f,w} & \longrightarrow & [S/h_0^k,F_0/h_1^\infty]_{s,f,w} & \xrightarrow{\theta} & [S/h_0^k,F_0/h_1^\infty]_{s+s_0,f+f_0,w+w_0} & \longrightarrow & [S/(h_0^k,\theta),F_0/h_1^\infty]_{s-1,f+1,w} \\
 & & \downarrow & & \downarrow & & \\
 & & [S,F_0/h_1^\infty]_{s,f,w} & \xrightarrow{P_r(-)} & [S,F_0/h_1^\infty]_{s+s_0,f+f_0,w+w_0} & &
\end{array}$$

- The vertical maps are isomorphisms when $f > \frac{1}{2}s + \frac{3}{2} - k$
- $[S/(h_0^k,\theta),F_0/h_1^\infty]_{s,f,w}$ admits a vanishing region of $f > \frac{1}{5}s + \frac{12}{5}$

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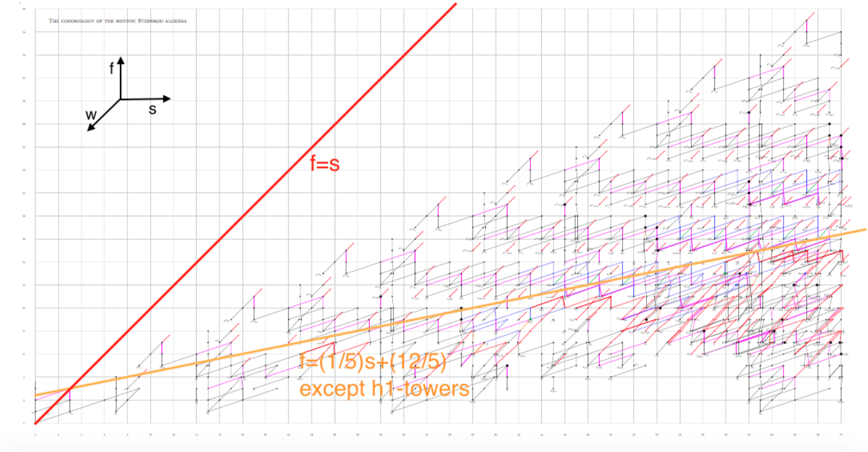

Figure: $\mathbb{C}$-motivic Ext

Thank you!