University of Kentucky
Department of Mathematics

MA 162 EXAM 2 FALL 2018

NAME: Solutions SECTION: ________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. We reserve the right to clear the memory on your calculator. Absolutely no cell phone use during the exam is allowed.

The exam consists of 16 multiple choice questions worth 5 points each and 2 short answer questions worth 10 points each. Record your answers to the Multiple Choice by filling in the single circle corresponding to the correct answer as shown below. In regards to the Multiple Choice portion, only this front page will be graded and no partial credit will be awarded.

(A) (B) (C) (D) (E)

All other work must be done in the body of the exam.

Multiple Choice Responses

Please indicate your answers for the multiple choice questions here by shading in your selections.

1	(A)	(B)	**(C)**	(D)	(E)	**9**	(A)	(B)	**(C)**	(D)	(E)
2	(A)	(B)	(C)	**(D)**	(E)	**10**	(A)	(B)	(C)	**(D)**	(E)
3	(A)	**(B)**	(C)	(D)	(E)	**11**	(A)	(B)	(C)	**(D)**	(E)
4	(A)	**(B)**	(C)	(D)	(E)	**12**	(A)	(B)	**(C)**	(D)	(E)
5	(A)	(B)	(C)	**(D)**	(E)	**13**	(A)	(B)	(C)	**(D)**	(E)
6	**(A)**	(B)	(C)	(D)	(E)	**14**	**(A)**	(B)	(C)	(D)	(E)
7	(A)	(B)	(C)	**(D)**	(E)	**15**	**(A)**	(B)	(C)	(D)	(E)
8	(A)	(B)	**(C)**	(D)	(E)	**16**	(A)	(B)	(C)	**(D)**	(E)

The following table is for administrative purposes only.

MC	17	18	Total
80	10	10	100

Multiple Choice Questions

Indicate your answer choices by **shading** in your answers on the **cover page**.

1. (5 points) If $X = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $Y = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, find $3X - 2Y$.

   A. $\begin{bmatrix} 1-a & 2-b \\ 3-c & 4-d \end{bmatrix}$

   B. $\begin{bmatrix} 3-2a & 2-b \\ 3-c & 4-d \end{bmatrix}$

   C. $\begin{bmatrix} 3-2a & 6-2b \\ 9-2c & 12-2d \end{bmatrix}$

   D. $\begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$

   E. None of the above are correct.

$$3X - 2Y = 3\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - 2\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 6 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$$
$$= \begin{bmatrix} 3-2a & 6-2b \\ 9-2c & 12-2d \end{bmatrix}$$

2. (5 points) If $X = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $Y = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, find the product XY.

   A. $\begin{bmatrix} a & 2b \\ 3c & 4d \end{bmatrix}$

   B. $\begin{bmatrix} a & 2c \\ 3b & 4d \end{bmatrix}$

   C. $\begin{bmatrix} 3-2a & 6-2b \\ 9-2c & 12-2d \end{bmatrix}$

   D. $\begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$

   E. None of the above are correct.

$$XY = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
$$= \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$$

3. (5 points) The dimension of a matrix X is 4×5, the dimension of a matrix Y is 3×7 and the dimension of a matrix Z is 5×6. Which of the following matrix products is possible?

   A. XY

   B. XZ

   C. ZX

   D. YZ

   E. ZY

To multiply an $n \times m$ and an $r \times s$ matrix, we need $m = r$

$X \cdot Z$ is a $(4\times\underbrace{5) \cdot (5}_{\text{Match}}\times 6)$

4. (5 points) If $X = \begin{bmatrix} 2 & b \\ 0 & 1 \end{bmatrix}$, find X^{-1}.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

A. $\begin{bmatrix} 1 & -b \\ 0 & 2 \end{bmatrix}$

B. $\frac{1}{2}\begin{bmatrix} 1 & -b \\ 0 & 2 \end{bmatrix}$

C. $\frac{1}{2}\begin{bmatrix} -2 & 0 \\ b & -1 \end{bmatrix}$

$$\begin{bmatrix} 2 & b \\ 0 & 1 \end{bmatrix}^{-1} = \frac{1}{2(1)-b(0)}\begin{bmatrix} 1 & -b \\ 0 & 2 \end{bmatrix}$$

D. $\begin{bmatrix} 2 & 0 \\ -b & 1 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} 1 & -b \\ 0 & 2 \end{bmatrix}$$

E. None of the above are correct.

5. (5 points) What is the result of applying the row operation $R_2 \to R_2 - 3R_1$ to the matrix

$$\begin{bmatrix} 1 & 5 & -1 \\ 3 & -2 & a \\ 1 & 2 & 3 \end{bmatrix}$$

$$\xrightarrow{R_2 \to R_2 - 3R_1} \begin{bmatrix} 1 & 5 & -1 \\ 0 & -17 & a+3 \\ 1 & 2 & 3 \end{bmatrix}$$

A. $\begin{bmatrix} 3 & 15 & -3 \\ 0 & -2 & a \\ 1 & 2 & 3 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 5 & -1 \\ 3 & -2 & a \\ 0 & -3 & 4 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 5 & -1 \\ 0 & -17 & a-3 \\ 1 & 2 & 3 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 5 & -1 \\ 0 & -17 & a+3 \\ 1 & 2 & 3 \end{bmatrix}$

E. None of the above are correct.

6. (5 points) Which of the following row operations is not allowed when using the Gauss-Jordan elimination method?

A. $R_1 \longrightarrow \frac{R_1}{R_2}$ → Can't divide one row by another

B. $R_1 \longrightarrow \frac{1}{3} \cdot R_1$ → Can multiply a row by nonzero constant

C. $R_1 \longleftrightarrow R_2$ → Can switch rows

D. $R_2 \longrightarrow R_2 - R_1$ → Can add a multiple of a row to another row

E. All of these row operations are allowed.

7. (5 points) Find a value k so that the system of equations corresponding to the following augmented matrix has infinitely many solutions. → One row is a multiple (or linear combination) of the other rows

$$\left[\begin{array}{ccc|c} 2 & -4 & 2 & 3 \\ 1 & -2 & 3 & 4 \\ -4 & 8 & -12 & k \end{array}\right]$$

$R_2(-4) \rightarrow \begin{bmatrix} 2 & -4 & 2 & 3 \\ -4 & 8 & -12 & -16 \\ -4 & 8 & -12 & k \end{bmatrix}$

A. $k = -6$

B. $k = 6$

C. $k = 16$

D. $k = -16$

E. None of these k values will give infinitely many solutions.

$\Rightarrow k = -16$ gives infinitely many solutions

8. (5 points) Determine x and y such that

$$\begin{bmatrix} 1 & 2 & 2 \\ -2 & 0 & -3 \end{bmatrix} + \begin{bmatrix} x-y & -1 & -2 \\ 3 & x & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 4 & -1 \end{bmatrix}$$

A. $(x, y) = (0, 0)$

B. $(x, y) = (4, -4)$

C. $(x, y) = (4, 4)$

D. $(x, y) = (0, 4)$

E. None of the above

$1 + x - y = 1 \Rightarrow x - y = 0$

$0 + x = 4 \Rightarrow x = 4$

$4 - y = 0$

$4 = y$

9. (5 points) Which of the following is the 2×2 **identity matrix**?

A. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ← Definition of identity 1's along the diagonal 0's elsewhere

D. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

E. None of the above are correct.

10. (5 points) The following augmented matrix is almost in reduced row echelon form. What is the solution to the corresponding system of equations?

$$\begin{array}{c} \begin{matrix} x & y & z & \end{matrix} \\ \left[\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 7 \end{array}\right] \end{array} \begin{array}{l} \to y=3 \\ \to z=4 \\ \to x=7 \end{array}$$

A. $(x, y, z) = (3, 4, 7)$
B. $(x, y, z) = (4, 3, 7)$
C. $(x, y, z) = (1, 1, 1)$
D. $(x, y, z) = (7, 3, 4)$
E. The system of equations has no solution.

11. (5 points) A system of equations with the same number of equations as variables can have

- I: A unique solution
- II: Infinitely many solutions
- III: No solution

Which choice describes the possible solutions to a system of equations with more variables than equations?

A. I
B. I and III
C. I, II, and III
D. II and III
E. None of the above.

To have a unique solution, need # of variables $\leq$ # of equations

12. (5 points) After performing row operations to an augmented matrix we arrive at the reduced row echelon form given below. What is the solution to the corresponding system of equations?

$$\begin{array}{c} \begin{matrix} x & y & z & \end{matrix} \\ \left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & 5 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$x+3z=2 \Rightarrow x=-3z+2$
$y+5z=4 \Rightarrow y=-5z+4$
$\leftarrow z$ is free variable

A. $(x, y, z) = (2, 4, 0)$
B. $(x, y, z) = (2z - 3, 4z - 5, z)$
C. $(x, y, z) = (2 - 3z, 4 - 5z, z)$
D. $(x, y, z) = (3, 5, 0)$
E. The system of equations has no solution.

13. (5 points) A theater has a seating capacity of 900 and charges \$3 for children, \$5 for students, and \$6 for adults. At a full capacity screening, there where half as many students as adults and children combined. The total money brought in was \$5000. Let x, y, z be the number of children, students, and adults who attended respectively. Which one of the following equations should not be used to determine the number of children, students, and adults that attended the show?

A. $x + y + z = 900$ → # of seats

B. $3x + 5y + 6z = 5000$ → \$ made

C. $y = \frac{1}{2}(x + z)$

D. $\frac{1}{2}y = x + z$

E. All of the above should be used to solve the system.

Not to be used

of ~~[illegible]~~ students = half the sum of adults and children

$$y = \frac{1}{2}(x+z)$$

14. (5 points) Consider a simple economy with two sectors; Sector 1 and Sector 2. Using an open Leontief model we are given the internal consumption (exchange) matrix, A, and external demand matrix, D. Assume the first row/column of A corresponds to Sector 1 and the second row/column corresponds to Sector 2. Let x be the output of Sector 1 and y be the output of Sector 2. Solve the open Leontief problem.

$$A = \begin{bmatrix} 0.5 & 0.2 \\ 0.5 & 0.6 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

A. $(x, y) = (14, 25)$

B. $(x, y) = (3, 2.5)$

C. $(x, y) = (1.6, 2.8)$

D. $(x, y) = (2, 5)$

E. None of the above

$$(I-A)^{-1}D = X$$

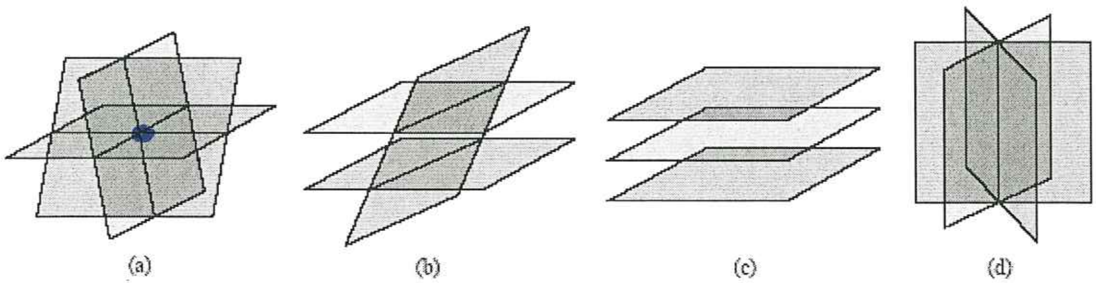
$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .5 & .2 \\ .5 & .6 \end{bmatrix}\right)^{-1}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} .5 & -.2 \\ -.5 & .4 \end{bmatrix}^{-1}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{.1}\begin{bmatrix} .4 & .2 \\ .5 & .5 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 5 & 5 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 14 \\ 25 \end{bmatrix}$$

15. (5 points) Below are four figures. Each of them corresponds to a system of 3 equations with 3 variables.



Which of the figures corresponds to an **independent** system of equations?

A. (a)
B. (b)
C. (c)
D. (d)
E. None of the above.

↑ exactly 1 point of intersection

16. (5 points) For what values of h and k will the following augmented matrix be inconsistent?

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 9 \\ 0 & 1 & 7 & 12 \\ 0 & 0 & h & k \end{array}\right]$$

A. $h \neq 0, k \neq 0$
B. $h \neq 0, k = 0$
C. $h = 0, k = 0$
D. $h = 0, k \neq 0$
E. None of the above.

Inconsistent means the last row gives a contradiction

If $h=0$ and $k \neq 0$ then we get $0 =$ non zero

Short Answer Questions

Show all work to receive credit for the following problems.

If you provide work on the scrap paper, indicate that within the body of the problem.

17. (15 points) A landlord owns 3 condominiums, a 1-bedroom condo, a 2-bedroom condo, and a 3 bedroom condo. The total rent she receives is \$5240. She needs to make repairs on the condos and it costs 10% of the 1-bedroom condo's rent for its repairs, 20% of the 2-bedroom condo's rent for its repairs, and 30% of the 3-bedroom condo's rent for its repairs. The total repair bill was \$276. The 3-bedroom repairs were twice as much as the 1-bedroom and 2-bedroom repairs combined. You would like to know the rent for all three condos.

   (a) Setup but **DO NOT SOLVE** this problem. Your answer should be a system of equations with **VARIABLES CLEARLY DEFINED**.

$x :=$ rent for 1-bedroom condo

$y :=$ rent for 2-bedroom condo

$z :=$ rent for 3-bedroom condo

$$x + y + z = 5240$$

$$.1x + .2y + .3z = 276$$

$$.3z = 2(.1x + .2y) \Rightarrow .2x + .4y - .3z = 0$$

   (b) Using your result from part a), create the augmented matrix corresponding to your system of equations. Your answer should be a matrix. **DO NOT REDUCE THE MATRIX**.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5240 \\ .1 & .2 & .3 & 276 \\ .2 & .4 & -.3 & 0 \end{array}\right]$$

18. (10 points) We assume that in a village there is a farmer, carpenter, and a tailor, who provide the three essential goods: food, shelter, and clothing. Suppose the farmer himself consumes 40% of the food he produces, and gives 30% to the carpenter, and 30% to the tailor. 23% of the carpenter's production is consumed by himself, 42% by the farmer, and 35% by the tailor. 52% of the tailor's production is used by himself, 33% by the farmer, and 15% by the carpenter. Write the closed Leontief model exchange matrix. Assume the first row and column represent the farmer, the second row and column represent the carpenter, and the third row and column represent the tailor. **DO NOT SOLVE THE PROBLEM, ONLY CREATE THE INTERNAL CONSUMPTION (EXCHANGE) MATRIX.**

$$\begin{bmatrix} .4 & .42 & .33 \\ .3 & .23 & .15 \\ .3 & .35 & .52 \end{bmatrix}$$