University of Kentucky
Department of Mathematics

MA 162 | EXAM 3 | FALL 2018

NAME: ______________________________ SECTION: __________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. We reserve the right to clear the memory on your calculator. Absolutely no cell phone use during the exam is allowed.

The exam consists of 16 multiple choice questions worth 5 points each and 3 short answer questions totaling 20 points. Record your answers to the Multiple Choice by filling in the single circle corresponding to the correct answer as shown below. In regards to the Multiple Choice portion, only this front page will be graded and no partial credit will be awarded.

● Ⓑ Ⓒ Ⓓ Ⓔ

All other work must be done in the body of the exam.

Multiple Choice Responses

Please indicate your answers for the multiple choice questions here by shading in your selections.

1 ● Ⓑ Ⓒ Ⓓ Ⓔ | 9 Ⓐ ● Ⓒ Ⓓ Ⓔ
2 Ⓐ Ⓑ Ⓒ ● Ⓔ | 10 ● Ⓑ Ⓒ Ⓓ Ⓔ
3 Ⓐ Ⓑ ● Ⓓ Ⓔ | 11 Ⓐ Ⓑ Ⓒ ● Ⓔ
4 ● Ⓑ Ⓒ Ⓓ Ⓔ | 12 ● Ⓑ Ⓒ Ⓓ Ⓔ
5 Ⓐ ● Ⓒ Ⓓ Ⓔ | 13 Ⓐ Ⓑ Ⓒ Ⓓ ●
6 Ⓐ Ⓑ ● Ⓓ Ⓔ | 14 Ⓐ Ⓑ Ⓒ ● Ⓔ
7 Ⓐ Ⓑ ● Ⓓ Ⓔ | 15 Ⓐ Ⓑ ● Ⓓ Ⓔ
8 Ⓐ Ⓑ Ⓒ ● Ⓔ | 16 Ⓐ Ⓑ ● Ⓓ Ⓔ

The following table is for administrative purposes only.

MC	17	18	19	Total
80	8	7	5	100

Multiple Choice Questions

Indicate your answer choices by **shading** in your answers on the **cover page**.

1. (5 points) Which of the following points is contained within the feasible region corresponding to the system of inequalities?

Only (2,0) satisfies both inequalities

$$3x + 6y < 12$$
$$5x + 7y > 8$$

A. (2,0)
B. (2,1)
C. (0,1)
D. (1,0)
E. None of the above points are in the feasible region.

2. (5 points) Consider the following maximization problem and select the correct number of **slack variables** required to solve the problem using the simplex method.

Maximize $P = x + 4y - 2z$

subject to

$$\left.\begin{aligned} x + 2y - 3z &\le 4 \\ 5x + 6y + 7z &\le 8 \\ 9x + 10y + 11z &\le 12 \\ 13x + 14y + 15z &\le 16 \end{aligned}\right\} \text{4 constraints}$$
$$x \ge 0, y \ge 0, z \ge 0$$

A. 3 slack variables
B. 1 slack variables
C. 7 slack variables
D. 4 slack variables
E. None of these.

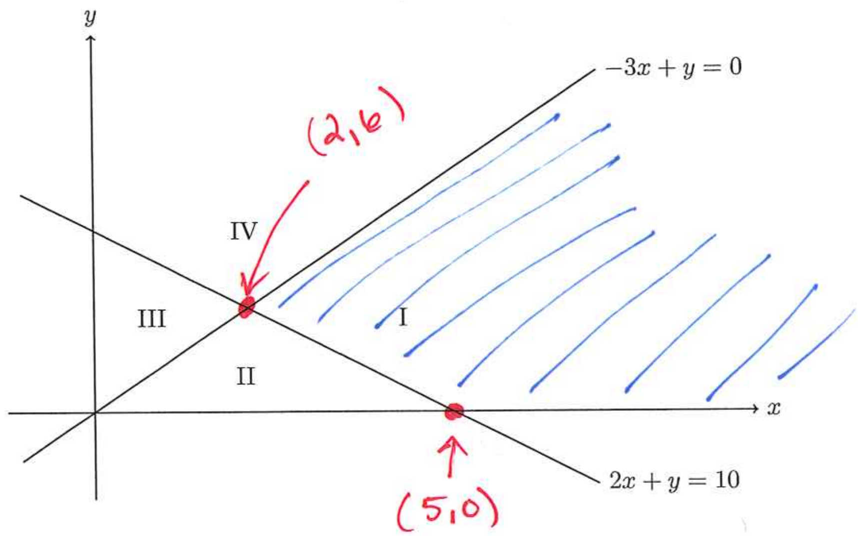
3. (5 points) When applying the simplex method to the simplex table

4	2	5	1	0	0	0	80
−2	−5	1	0	1	0	0	10
7	3	−3	0	0	1	0	21
−9	−18	−12	0	0	0	1	0

Ratio
80/2=40
21/3=7 ← smallest positive ratio

which row should be selected to perform the pivot operation? Note: you must choose the correct column first.

A. R_1
B. R_2
C. R_3
D. R_4
E. None of these.



4. (5 points) Consider the feasible region given by the following inequalities, whose boundary lines are graphed above.

$$-3x+y \le 0 \qquad 2x+y \ge 10 \qquad x \ge 0 \qquad y \ge 0$$

1st quadrant

Which one of the following labels best indicate the feasible region described above?

$y \le 3x$ $\quad$ $y \ge -2x+10$

A. I
B. II
C. III
D. IV
E. None of these labels adequately or completely indicate the feasible region.

5. (5 points) At which point is the function $F = 3x + 4y$ **minimized** with respect to the feasible region described in problem 4 above?

A. $(0,0)$
B. $(5,0)$
C. $(2,6)$
D. $(0,10)$
E. None of these or no solutions exist.

x	y	F=3x+4y
5	0	15
2	6	30

6. (5 points) A maximization linear programming problem has been solved by the simplex method and the final simplex table is

x	y	z	u	v	w	P	Constant	
1	1	0	.5	0	0	0	8	← y = 8
0	0	0	−3	1	1	0	3	
1	0	1	1	0	4	0	2	← z = 2
0	0	0	2	0	3	1	7	

Which of the following is an optimal point indicated by the table?

x, u, w = 0

A. $(x, y, z) = (8, 3, 2)$
B. $(x, y, z) = (8, 2, 3)$
C. $(x, y, z) = (0, 8, 2)$
D. $(x, y, z) = (0, 0, 2)$
E. None of these.

7. (5 points) When graphing the feasible region for the given system of inequalities, which point can **NOT** be used as a test point?

$$-3x + y \le 6$$
$$2x + 5y \le 10$$

Test points can not be on either line.

(0,6) is on
$-3x+y=6$

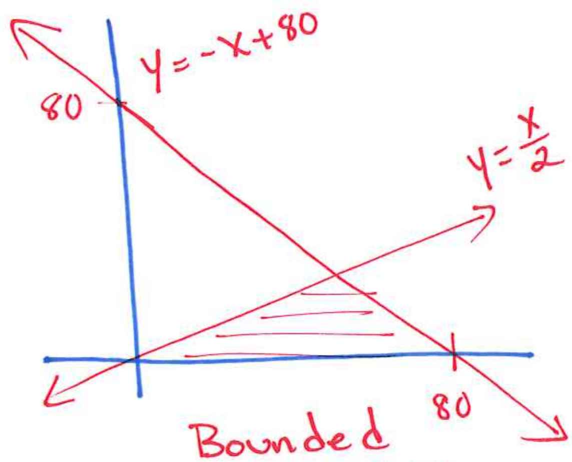
A. $(2, -5)$
B. $(-2, -1)$
C. $(0, 6)$
D. $(4, -2)$
E. All of these points can be used as test points.

8. (5 points) Consider the linear programming problem below and statements I, II and III that follow.

Optimize $F = 31x + 55y$ subject to

$$\begin{array}{rcrcr} x & + & y & \le & 80 \\ x & - & 2y & \ge & 0 \end{array}$$

and $x \ge 0$, $y \ge 0$.



Which of the following statements are true?

I. F must have a minimum on the given feasible region.
II. The feasible region is bounded.
III. F must have a maximum on the given feasible region.

Bounded
⇒ Min & Max

A. Only II and III are true
B. Only I is true.
C. Only I and II are true
D. I, II, and III are all true
E. None of the answer choices provided above are fully accurate.

9. (5 points) A minimization linear programming problem has been converted to its dual maximization problem. The simplex method was then applied to the maximization problem. The final maximization simplex table is below.

u	v	x	y	z	P	Constant
1	0	.5	4	1	0	8
0	0	-3	1	5	0	3
0	1	1	3	4	0	2
0	0	22	15	17	1	300

Which of the following is the point at which the minimization problem is optimized (minimized)?

A. $(u, v) = (8, 2)$
B. $(x, y, z) = (22, 15, 17)$
C. $(x, y, z) = (8, 3, 2)$
D. $(x, y, z) = (2, 3, 8)$
E. None of the above

Read the solution to the minimization problem from the bottom row of the dual maximization problem.

10. (5 points) Beyer Pharmaceutical produces three kinds of cold formulas: I, II, and III. It takes 1.5 hr to produce 1000 bottles of Formula I, 2 hr to produce 1000 bottles of Formula II, and 3 hr to produce 1000 bottles of Formula III. The profits for each 1000 bottles of Formula I, Formula II, and Formula III are \$100, \$150, and \$210, respectively. Suppose that for a certain production run, there are enough ingredients on hand to make at most 7000 bottles of Formula I, 14000 bottles of Formula II, and 3000 bottles of Formula III. Furthermore, suppose the time for the production run is limited to a maximum of 50 hr. They want to maximize their profit, P, in dollars. Which of the following is the correct objective function?

A. $P = 100x + 150y + 210z$
B. $P = 1000x + 1000y + 1000z$
C. $P = 7000x + 140000y + 3000z$
D. $P = 1.5x + 2y + 3z$
E. None of these are the objective function.

Assume X = # of 1000 bottle boxes of formula I
Y = " " formula II
Z = " " formula III

11. (5 points) The following maximization problem is not in standard form. Which of the following changes would make the linear programming problem a standard form maximization problem?

$$\begin{array}{rl} \text{Maximize} & P = 2x + 9y \\ \text{Subject to} & 3x + 5y \geq 3 \\ & 9x + 5y \leq 8 \\ & x \geq 0, y \geq 0 \end{array}$$

$\longrightarrow$ need $3x+5y \leq 3$

A. Change $9x + 5y \leq 8$ to $9x + 5y \geq 8$
B. Change Maximize $P = 2x + 9y$ to Minimize $P = 2x + 9y$
C. Remove the conditions $x \geq 0, y \geq 0$
D. Change $3x + 5y \geq 3$ to $3x + 5y \leq 3$
E. None of the above changes would allow us to use the simplex method.

12. (5 points) While performing the simplex method, there are 2 positive entries in the bottom row left of the vertical line in the simplex table. The rest of the entries in the bottom row are zero. How many more iterations of the simplex method should be performed to find the optimal solution?

A. 0
B. 1
C. 2
D. 3
E. The number of iterations can not be determined from this information

Once there are no negative ~~[illegible]~~ entries in the bottom row, the simplex algorithm is complete.

13. (5 points) After completing the simplex method, the solution can be read from the final simplex table. Which variables are **NOT** automatically set equal to zero?

A. Slack Variables
B. Non-basic Variables
C. Independent Variables
D. Dependent Variables
E. Basic Variables

14. (5 points) One constraint for a standard maximization problem is

$$x \leq .2\,(x+y)$$

What is the correct way to rearrange this as an equation (including a slack variable u) before entering it into a Simplex Table?

A. $-.8x + .2y + u = 0$
B. $-1.2x + .2y + u = 0$
C. $1.2x - .2y + u = 0$
D. $.8x - .2y + u = 0$
E. None of the above.

$x \leq .2x + .2y$

$.8x - .2y \leq 0 \leftarrow$ standard form

Add slack variable u to get

$.8x - .2y + u = 0$

15. (5 points) Consider the following minimization problem.

$$\begin{array}{ll} \text{Minimize} & P = 2x + 9y \\ \text{Subject to} & 3x + 5y \geq 3 \\ & 9x + 5y \geq 8 \\ & x \geq 0, y \geq 0 \end{array}$$

What is the objective function of the dual maximization problem? Use u and v for the dual variables.

A. $C = 9u + 3v$

B. $C = 3u + 9v$

C. $C = 3u + 8v$

D. $C = 5u + 5v$

E. None of the above.

$$\left[\begin{array}{cc|c} 3 & 5 & 3 \\ 9 & 5 & 8 \\ \hline 2 & 9 & 0 \end{array}\right] \xrightarrow{\text{transpose}} \left[\begin{array}{cc|c} 3 & 9 & 2 \\ 5 & 5 & 9 \\ \hline 3 & 8 & 0 \end{array}\right]$$

$C = 3u + 8v$

16. (5 points) Apply the simplex method to the following simplex table which corresponds to a standard maximization problem. At what point is the objective function maximized?

pivot entry

x	y	u	v	P	Constant
$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	11000
2	0	$\frac{2}{3}$	$-\frac{4}{3}$	0	16000
-1	0	0	3	2	30000

Ratio

$11000/\frac{1}{2} = 22000$

$16000/2 = 8000$

smallest ratio

A. $(x, y) = (11000, 16000)$

B. $(x, y) = (0, 11000)$

C. $(x, y) = (8000, 7000)$

D. $(x, y) = (7000, 8000)$

E. None of these points gives the correct maximum.

$R_2 \to \frac{1}{2}R_2$
$$\left[\begin{array}{ccccc|c} \frac{1}{2} & 1 & 0 & \frac{1}{2} & 0 & 11000 \\ 1 & 0 & \frac{1}{3} & -\frac{2}{3} & 0 & 8000 \\ \hline -1 & 0 & 0 & 3 & 2 & 30000 \end{array}\right]$$

$R_1 \to R_1 - \frac{1}{2}R_2$

$R_3 \to R_3 + R_2$
$$\left[\begin{array}{ccccc|c} 0 & 1 & -1/6 & 5/6 & 0 & 7000 \\ 1 & 0 & \frac{1}{3} & -\frac{2}{3} & 0 & 8000 \\ \hline 0 & 0 & \frac{1}{3} & \frac{7}{3} & 2 & 38000 \end{array}\right]$$

$y = 7000$

$x = 8000$

Short Answer Questions

Show all work to receive credit for the following problems.
If you provide work on the scrap paper, indicate that within the body of the problem.

17. (8 points) **SETUP BUT DO NOT SOLVE** the following linear programming problem. A complete answer to this problem must include

- clearly defined variables,
- a clearly labeled objective function, and
- all necessary constraints.

You should **NOT** setup a simplex table.

Oily Oil Company has decided to introduce three oil mixes made from blending two or more oils. One jar of olive-vegetable oil requires 6 oz each of olive and vegetable oils. One jar of vegetable-peanut oil requires 10 oz of vegetable oil and 6 oz of peanut oil. Finally, one jar of olive-vegetable-peanut oil requires 3 oz of olive oil, 7 oz of vegetable oil, and 2 oz of peanut oil. The company has decided to allot 15,000 oz of olive oil, 23,000 oz of vegetable oil, and 4000 oz of peanut oil for the initial production run. Its profit on one jar of olive-vegetable oil is \$1.10, its profit on one jar of vegetable-peanut oil is \$0.70, and its profit on one jar of olive-vegetable-peanut oil is \$0.60. To realize a maximum profit, how many jars of each blend should the company produce?

x = # of jars of olive-vegetable oil produced
y = " " vegetable-peanut " "
z = " " olive-vege-peanut " "

Objective: Maximize Profit

$$P = 1.1x + .7y + .6z$$

Constraints:

$$6x + 3z \le 15000 \quad \text{(olive)}$$
$$6x + 10y + 7z \le 23000 \quad \text{(vegetable)}$$
$$6y + 2z \le 4000 \quad \text{(peanut)}$$

$x, y, z \ge 0$ ← must include non-negativity constraints

18. (7 points) Consider the given standard maximization problem

Maximize	$P = 3x + 5y + 7z$
subject to	$3x + 4y - 5z \leq 24$
	$5x + 2y + 8z \leq 10$
	$x \geq 0, y \geq 0, z \geq 0$

Set up the simplex table corresponding to the maximization problem. **LABEL EACH COLUMN** with the variable corresponding to the entries in that column.

DO NOT PERFORM THE SIMPLEX ALGORITHM.

Let u,v be slack variables

$\Rightarrow$ $3x + 4y - 5z + u = 24$

$5x + 2y + 8z + v = 10$

$x, y, z, u, v \geq 0$, $P - 3x - 5y - 7z = 0$

x	y	z	u	v	P	constant
3	4	-5	1	0	0	24
5	2	8	0	1	0	10
-3	-5	-7	0	0	1	0

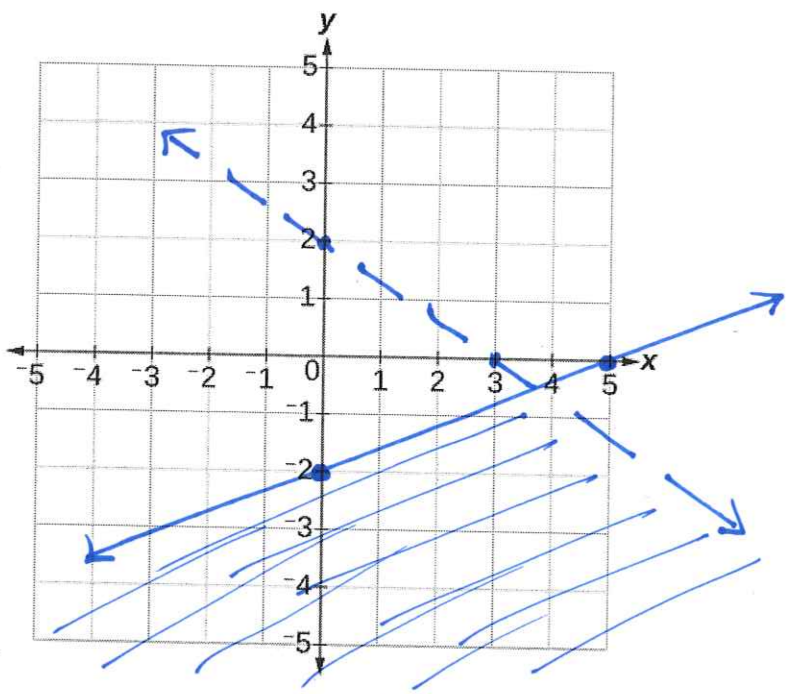
19. (5 points) Determine graphically the feasible region for the system of inequalities

$$2x - 5y \geq 10$$
$$4x + 6y < 12$$

Note: A complete answer to this problem must include the following.

- x and y intercepts of each line entered in the given table written as points $(x, 0)$ and $(0, y)$.
- a clearly drawn and labeled feasible region on the provided axes.

Line	x-intercept	y-intercept
$2x - 5y = 10$	$(5,0)$	$(0,-2)$
$4x + 6y = 12$	$(3,0)$	$(0,2)$



Use $(0,0)$ as test point

$2(0) - 5(0) \geq 10$ False (shade on opposite side)

$4(0) + 6(0) < 12$ True (Shade on same side)

OR $\rightarrow$ $2x - 5y \geq 10 \Rightarrow y \leq \frac{2}{5}x - 2$ (below)

$4x + 6y < 12 \Rightarrow y < -\frac{2}{3}x + 2$ (below)