

**University of Kentucky  
Department of Mathematics**

MA 162

Solutions

EXAM 4

FALL 2018

NAME: \_\_\_\_\_ SECTION: \_\_\_\_\_

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS) or a QWERTY keyboard is permitted. We reserve the right to clear the memory on your calculator. Absolutely no cell phone use during the exam is allowed.

The exam consists of 16 multiple choice questions worth 5 points each and 2 short answer questions worth 10 points each. Record your answers to the Multiple Choice by filling in the single circle corresponding to the correct answer as shown below. In regards to the Multiple Choice portion, only this front page will be graded and no partial credit will be awarded.

● (B) (C) (D) (E)

All other work must be done in the body of the exam.

**Multiple Choice Responses**

Please indicate your answers for the multiple choice questions here by shading in your selections.

- |   |  |
|---|--|
| <p>1    <input checked="" type="radio"/> (A)    (B)    (C)    (D)    (E)</p> <p>2    (A)    (B)    (C)    <input checked="" type="radio"/> (D)    (E)</p> <p>3    (A)    (B)    <input checked="" type="radio"/> (C)    (D)    (E)</p> <p>4    (A)    <input checked="" type="radio"/> (B)    (C)    (D)    (E)</p> <p>5    (A)    <input checked="" type="radio"/> (B)    (C)    (D)    (E)</p> <p>6    (A)    (B)    (C)    <input checked="" type="radio"/> (D)    (E)</p> <p>7    <input checked="" type="radio"/> (A)    (B)    (C)    (D)    (E)</p> <p>8    (A)    (B)    (C)    <input checked="" type="radio"/> (D)    (E)</p> | <p>9    (A)    <input checked="" type="radio"/> (B)    (C)    (D)    (E)</p> <p>10    (A)    (B)    <input checked="" type="radio"/> (C)    (D)    (E)</p> <p>11    <input checked="" type="radio"/> (A)    (B)    (C)    (D)    (E)</p> <p>12    (A)    (B)    (C)    <input checked="" type="radio"/> (D)    (E)</p> <p>13    <input checked="" type="radio"/> (A)    (B)    (C)    (D)    (E)</p> <p>14    (A)    (B)    (C)    <input checked="" type="radio"/> (D)    (E)</p> <p>15    (A)    (B)    (C)    <input checked="" type="radio"/> (D)    (E)</p> <p>16    <input checked="" type="radio"/> (A)    (B)    (C)    (D)    (E)</p> |
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The following table is for administrative purposes only.

MC	17	18	Total
80	10	10	100

## Multiple Choice Questions

Indicate your answer choices by shading in your answers on the cover page.

1. (5 points) Find the present value of an investment if the future value is \$3575.50 and the investment earns 7.46% APR for 5 years compounded semi-annually.

- A. \$2479.09  
 B. \$2495.19  
 C. \$4671.77  
 D. \$5123.55  
 E. None of the above

$$A = P \left(1 + \frac{r}{m}\right)^{mt}$$

$$3575.50 = P \left(1 + \frac{0.0746}{2}\right)^{2 \cdot 5}$$

$$3575.50 = P(1.442260731)$$

$$\Rightarrow P = 2479.09$$

2. (5 points) Find the future value of an investment if the present value is \$1892.06 and the investment earns 4.51% APR for 7 years compounded continuously.

- A. \$1379.84  
 B. \$7703.69  
 C. \$2532.93  
 D. \$2594.43  
 E. None of the above

$$A = Pe^{rt}$$

$$A = 1892.06 e^{(0.0451)(7)}$$

$$A = 2594.43$$

3. (5 points) Find the periodic payment of a 8-year annuity if the future value is \$16181.51 and the annuity earns 3.22% APR compounded quarterly.

- A. \$574.49  
 B. \$575.62  
 C. \$445.36  
 D. \$678.18  
 E. None of the above

$$F = R \left[ \frac{\left(1 + \frac{r}{m}\right)^{mt} - 1}{\frac{r}{m}} \right]$$

$$16181.51 = R \left[ \frac{\left(1 + \frac{0.0322}{4}\right)^{4(8)} - 1}{\frac{0.0322}{4}} \right]$$

$$16181.51 = R[36.33385652] \Rightarrow R = 445.36$$

4. (5 points) Find the present value of a 6-year annuity if the periodic payment is \$114.15 and the annuity earns 3.75% APR compounded quarterly.

- A. \$3056.31  
 B. \$2443.07  
 C. \$2477.31  
 D. \$5111.80  
 E. None of the above

$$P = R \left[ \frac{1 - \left(1 + \frac{r}{m}\right)^{-mt}}{\left(\frac{r}{m}\right)} \right]$$

$$P = 114.15 \left[ \frac{1 - \left(1 + \frac{0.0375}{4}\right)^{-4(6)}}{\left(\frac{0.0375}{4}\right)} \right]$$

$$P = 2443.07$$

5. (5 points) Find the effective interest rate of an investment which earns 5.98% APR compounded semi-annually.

- A. 5.99%  
 B. 6.07%  
 C. 5.27%  
 D. 4.03%  
 E. None of the above

Find the value of \$1 after 1 year and subtract 1

$$\text{Eff Rate} = 1 \left(1 + \frac{0.0598}{2}\right)^{2 \cdot 1} - 1$$

$$= 0.06069401$$

$$\Rightarrow 6.07\%$$

6. (5 points) Find the balance of an 11-year loan after 4 years if the original amount of \$19908.01 was borrowed at 7.7% APR compounded monthly.

- A. \$6292.15  
 B. \$19011.79  
 C. \$15809.62  
 D. \$14514.05  
 E. None of the above

First, find the payment.

$$19908.01 = R \left[ \frac{1 - \left(1 + \frac{0.077}{12}\right)^{-12(11)}}{\frac{0.077}{12}} \right]$$

$$\Rightarrow R = 224.055877$$

Now, find the value of the loan with 7 years left in the loan.

$$P = 224.055877 \left[ \frac{1 - \left(1 + \frac{0.077}{12}\right)^{-12(7)}}{\frac{0.077}{12}} \right]$$

$$\Rightarrow P = 14514.05061$$

7. (5 points) How many years does it take a lump sum investment of \$4995.02 to grow to \$7177.57 if the investment earns 8.55% APR compounded continuously.

- A. 4.24 years  
 B. 3.10 years  
 C. 4.13 years  
 D. 2.09 years  
 E. None of the above

$$A = Pe^{rt}$$

$$7177.57 = 4995.02 e^{(.0855)t}$$

$$\ln(1.436945197) = \ln(e^{.0855t})$$

$$\ln(1.436945197) = .0855t$$

$$\frac{\ln(1.436945197)}{.0855} = t$$

$$\Rightarrow t = 4.24$$

8. (5 points) It takes 5 years for a lump sum investment to grow from \$5000 to a total of \$7240. Assuming the interest is compounded quarterly, at what annual interest rate was the money invested?

- A. 29.88%  
 B. 4.29%  
 C. 1.85%  
 D. 7.47%  
 E. None of the above

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

$$7240 = 5000 \left(1 + \frac{r}{4}\right)^{4(5)}$$

$$(1.448)^{\frac{1}{20}} = \left(1 + \frac{r}{4}\right)^{\frac{1}{20}}$$

$$1.018681521 = 1 + \frac{r}{4}$$

$$.018681521 = \frac{r}{4}$$

$$.074726 = r \Rightarrow r = 7.47\%$$

9. (5 points) An amount of \$8,000 is borrowed at a discount rate of 12%, find the proceeds if the length of the loan is 7 months.

- A. \$560  
 B. \$7440  
 C. \$1280  
 D. \$6720  
 E. None of the above

$$P = M(1 - rt)$$

$$P = 8000 \left(1 - (.12)\left(\frac{7}{12}\right)\right)$$

$$P = 7440$$

10. (5 points) Over the course of the last year, Kyle's investment account has grown by 4.3%. Currently, Kyle has \$1,564.50 in this account. What was the balance in his account one year ago, before this gain?

- A. \$1631.77  
 B. \$1497.23  
 C. \$1500.00  
 D. \$1234.54  
 E. None of the above

$$A = P(1 + rt)$$

$$1564.50 = P(1 + (.043)(1))$$

$$\frac{1564.50}{1.043} = P$$

$$\Rightarrow P = 1500$$

11. (5 points) An investment pays simple interest, and quadruples in 17 years. What is the annual interest rate?

- A. 17.65%  
 B. 8.15%  
 C. 23.53%  
 D. 4.24%  
 E. None of the above

$$A = P(1 + rt)$$

$$4P = P(1 + r(17))$$

$$4 = 1 + 17r$$

$$3 = 17r$$

$$r = \frac{3}{17} = .17647$$

12. (5 points) If you make quarterly deposits of \$425 for 5 years into an ordinary annuity earning an annual interest rate of 3.08%, how much interest did you earn in those 5 years?

- A. \$9151.46  
 B. \$650.07  
 C. \$7849.92  
 D. \$651.46  
 E. None of the above

First find future value of the annuity.

$$F = 425 \left[ \frac{(1 + \frac{.0308}{4})^{4(5)} - 1}{\frac{.0308}{4}} \right]$$

$$\Rightarrow F = 9151.464679$$

Now subtract the total payments.

$$\text{Total} = 425(4)(5) = 8500$$

$$\Rightarrow \text{Interest} = 9151.46 - 8500 = 651.46$$

13. (5 points) A couple has decided to purchase a \$200,000 house using a down payment of \$30,000. They can pay off the balance of the loan at 10% over 30 years making monthly payments. How much, including the down payment, have they paid toward the purchase price of the house after 5 years? In other words, find their equity in the house after 5 years.

- A. \$35,823.66  
 B. \$36,851.36  
 C. \$99,784.51  
 D. \$27,504.92  
 E. None of the above

$$\text{Loan amount} = 200000 - 30000 = 170000$$

Find Payment.

$$170000 = R \left[ \frac{1 - \left(1 + \frac{.10}{12}\right)^{-12(30)}}{\frac{.10}{12}} \right]$$

$$\Rightarrow R = 1491.871669$$

Find balance of loan with 25 years left

$$P = 1491.871669 \left[ \frac{1 - \left(1 + \frac{.10}{12}\right)^{-12(25)}}{\frac{.10}{12}} \right] \Rightarrow P = 164176.3448$$

$\Rightarrow$  They have paid  $170000 - 164176.3448 = 5823.66$

14. (5 points) Bob makes his first \$850 deposit into an IRA earning 6% compounded annually on the day he turns 21 and his last \$850 deposit on the day he turns 47 (27 equal deposits in all.) With no additional deposits, the money in the IRA continues to earn 6% interest compounded monthly for another 18 years until Bob retires. How much is the IRA worth when Bob retires?

- A. \$154,562.18  
 B. \$32,051.24  
 C. \$32,976.80  
 D. \$159,025.59  
 E. None of the above

Equity =  $30000 + 5823.66$

After 27 years

$$F = 850 \left[ \frac{\left(1 + \frac{.06}{1}\right)^{1(27)} - 1}{\frac{.06}{1}} \right] = 54149.90$$

Now use lump sum formula  $A = P \left(1 + \frac{r}{m}\right)^{mt}$  for remaining 18 years.

$$A = 54149.90 \left(1 + \frac{.06}{12}\right)^{12(18)} = 159025.59$$

*Fixed*

15. When considering a present value in an account that earns a positive interest rate, which of the following, if any, will decrease the future value of the investment?

- (I) Increasing the interest rate
  - (II) Increasing the amount of time the investment is left in the account
  - (III) Decrease the number of compounds per year
- A. Only I
  - B. Only II
  - C. Both I and II
  - D. Only III
  - E. None of the above answer choices are fully correct

*Increasing the rate, time, or # of compounds increases the future value.*

16. (5 points) Irene plans to retire in 30 years. She has been preparing to retire by making annual deposits of \$5000 into an account that pays an annual interest rate of 6.3%. She continued this practice every year for 20 years. Her goal is to have \$500000 saved up at the time of her retirement 10 years from now. How large should her next 10 annual deposits be, at the same interest rate of 6.3%, so that she can reach her goal?

- A. \$11223.73
- B. \$23191.90
- C. \$6245.76
- D. \$3418.26
- E. None of the above

Now

*Make payments* → 
$$F = 5000 \left[ \frac{(1 + \frac{.063}{1})^{1(20)} - 1}{\frac{.063}{1}} \right]$$

*20 years*

*30 years*

$= 189971.1311$

*use  $A = P(1 + \frac{r}{m})^{mt}$  to move forward 10 years*

$A = 189971.13 \left( 1 + \frac{.063}{1} \right)^{1(10)}$

$= 349961.4875$

*subtract from 500 000*

$500000 - 349961.4875 = 150038.5125$

*Future value of new plan*

Find Payment

$150038.5125 = R \left[ \frac{(1 + \frac{.063}{1})^{1(10)} - 1}{\frac{.063}{1}} \right]$

$\Rightarrow R = 11223.73$

**Short Answer Questions**

Show all work to receive credit for the following problems.

If you provide work on the scrap paper, indicate that within the body of the problem.

17. (10 points) During the last 30 years that George has worked for Mapple Incorporated he has deposited \$250 per month into his retirement account earning 2.3% APR compounded monthly. Now George is going to retire. How much money does George have in his retirement account?

$$F = R \left[ \frac{\left(1 + \frac{r}{m}\right)^{mt} - 1}{\frac{r}{m}} \right]$$

$$F = 250 \left[ \frac{\left(1 + \frac{.023}{12}\right)^{12(30)} - 1}{\frac{.023}{12}} \right]$$

$$= \underline{\underline{\$129,443.39}}$$



18. (10 points) After 40 years of working for Omnivox, Hank has decided to retire. Hank has saved \$800,000 in his retirement account that earns 1.6% APR compounded quarterly. He would like to withdraw money from the account over the next 11 years each quarter. What is the largest regular amount Hank can withdraw from the account each quarter for the next 11 years?

*extra information*

$$P = R \left[ \frac{1 - \left(1 + \frac{r}{m}\right)^{-mt}}{\frac{r}{m}} \right]$$
$$800000 = R \left[ \frac{1 - \left(1 + \frac{.016}{4}\right)^{-4(11)}}{\frac{.016}{4}} \right]$$

$$\Rightarrow R = \$19864.97$$