1. Consider the following Maple session
   > 3^2;
   > 4^2;
   > % + %%;
   16
   does the last instruction make sense? If so, what is the result? If not, why?
   > 3^2;
   > 4^2;
   > % + %%;
   16
   25
   It makes sense because *Maple* does not need to have the result printed to screen in order to know the
   outcome of a command.

2. Explain the different results of the following *Maple* commands.
   (a) \texttt{x:y;}
   (b) \texttt{x/y;}
   (c) \texttt{x\y}
      > x:y;
      y
   \texttt{Maple} simply returns \( y \). Putting the colon behind \( x \) tells \texttt{Maple} to execute that statement but not
   print any output.
   > x/y;
   \[
   \frac{x}{y}
   \]
   This gives the fraction \( x/y \).
   > x\y;
   \[
   xy
   \]
   The backslash, \( \backslash \), is a continuation character.

3. In this exercise you can practice your skills in using the help system of \texttt{Maple}. 
(a) Suppose that you can want to select from an equation, e.g., \(1 = \cos(x)^2 + \sin(x)^2\), only the left or right side. How can you easily do this in Maple?

\[
\text{eqn := 1 = cos(x)^2 + sin(x)^2}
\]

\[
\text{eqn := 1 = cos(x)^2 + sin(x)^2}
\]

\[
\text{lhs(eqn)};
\]

\[
1
\]

\[
\text{rhs(eqn)};
\]

\[
\cos(x)^2 + \sin(x)^2
\]

(b) Suppose that you want to compute the continued fraction approximation of the exponential function; can Maple do this for you? If yes, carry out the computation.

\[
\text{convert(exp(x),confrac,x)};
\]

\[
1 + \frac{x}{1 + \frac{-2x}{1 + \frac{-3x}{2 + \frac{1}{5}x}}}
\]

(c) Suppose that you want to factor the polynomial \(x^8 + x^6 + 10 x^3 + 8 x^2 + 2 x + 8\) modulo 13. Can Maple do this? If yes, carry out this factorization.

\[
\text{Factor(x^8+x^6+10*x^3+8*x^2+2*x+8)mod 13};
\]

\[
(x^2 + 8 x + 9) (x + 7) (x^2 + 11 x + 12) (x^3 + 6 x + 4)
\]

(d) Suppose that you want to determine all subsets of the set \(\{1, 2, 3, 4, 5\}\). How can you do this in Maple?

\[
\text{powerset}\;
\]

\[
\{\{\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{2, 5\}, \{3, 4\}, \{3, 5\}, \{4, 5\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 2, 5\}, \{1, 3, 4\}, \{1, 3, 5\}, \{1, 4, 5\}, \{2, 3, 4\}, \{2, 3, 5\}, \{2, 4, 5\}, \{3, 4, 5\}\}
\]

4. Load the `numtheory` package by entering `with(numtheory);` You may recognize some functions from number theory; some of the routines in this package are useful in answering the following questions.

\[
\text{divisors(9876543210123456789)};
\]
\{1, 3, 9, 13, 39, 117, 6353, 8969, 19059, 26907, 57177, 80721, 82589, 116597, 247767, 349791, 743301, 1049373, 2222222223, 253244697695473251, 759734093086419753, 172736296240157, 3292181070041152263, 518208888720471, 154626666161413, 39862222209267, 6666666669, 119586666627801, 1097393690013717421, 13287407403089, 9411851848793, 9876543210123456789, 44444444444, 173333333277, 57777777759, 19259259253, 13333333329, 367062222102927, 110118666308781, 84414899231824417, 28235555546379, 84706666639137, 122354074034309, 1481481481, 56980057, 170940171, 512820513, 740740741\} 

(b) Find the prime number that is closest to 9,876,543,210,123,456,789.

\[
\text{nextprime}(9876543210123456789) = 9876543210123456803 \\
\text{prevprime}(9876543210123456789) = 9876543210123456781 \\
\%\% - 9876543210123456789 = 14 \\
9876543210123456789 - \%\% = 8
\]

The closest prime to 9,876,543,210,123,456,789 is 9,876,543,210,123,456,781.

(c) What is the prime factorization of \(5^{(5^{(5^5)})}\)?

\[
\text{ifactor}(5^{(5^{(5^5)})})
\]

Error, numeric exception: overflow

(d) Expand the base \(e\) of the natural logarithm as a continued fraction up to 10 levels deep.

\[
\text{cfrac}(\exp(1), 9) = \frac{2}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \cdots}}}}}}}}}
\]

5. In Maple, what is the difference between \(\frac{1}{3} + \frac{1}{3} + \frac{1}{3}\) and \(\frac{1.0}{3.0} + \frac{1.0}{3.0} + \frac{1.0}{3.0}\)?

\[
1/3 + 1/3 + 1/3 = 1.0/3.0 + 1.0/3.0 + 1.0/3.0
\]

How many digits of accuracy are necessary to make Maple think that they are the same, or will it ever consider them to be the same?

\[
\text{Digits:=1000;}
\]

\[
1.0/3.0 + 1.0/3.0 + 1.0/3.0
\]

\[0.999999999\]
6. Find the floating-point approximation of \((e^{\pi \sqrt{163}})^{3/2}\) using a precision of 10, 20, and 30 digits, respectively.

\[
\begin{align*}
\text{restart;} \\
\text{evalf(exp(Pi*sqrt(163)/3),10);} & \quad \text{#10 digit precision} \\
& 640319.9998 \\
\text{evalf(exp(Pi*sqrt(163)/3),20);} & \quad \text{#20 digit precision} \\
& 640320.00000000060543 \\
\text{evalf(exp(Pi*sqrt(163)/3),30);} & \quad \text{#30 digit precision} \\
& 640320.000000000604863735049036
\end{align*}
\]

7. Calculate \(\pi^{\pi^\pi}\) to nine decimal places.

First, we expect that we should simply execute

\[
\begin{align*}
\text{evalf(Pi^(Pi^Pi),9);} \\
& .13401641830063574352974491296610^{19}
\end{align*}
\]

but this only gives 9 digits, NOT 9 decimal places. We need a few more digits to get 9 decimal places.

\[
\begin{align*}
\text{evalf(Pi^(Pi^Pi),30);} \\
& .134016418306357435297491296610^{19}
\end{align*}
\]

8. Compute this exercise in a floating-point precision of eight decimal places. What is the result of \(310.0 \times 320.0 \times 330 - \sqrt{310.0} \times 320.0 \times 330.0 \times 330.0 \times 310.0\)?

\[
\begin{align*}
\text{Digits:=8;} \\
\text{310.0*320.0*330 - sqrt(310.0)*320.0*330.0*330.0*310.0);} \\
& -1.
\end{align*}
\]

\[
\begin{align*}
\text{restart;
\end{align*}
\]
9. Do you remember which of the numbers $\frac{19}{6}$, $\frac{22}{7}$, and $\frac{25}{8}$ is a fairly good rational approximation of $\pi$? Use Maple to find the best of these three numbers. Find the best rational approximation $\frac{a}{b}$ of $\pi$, where $a$ and $b$ are natural numbers less than 1000 (Hint: look at the continued fraction expansion of $\pi$).

```maple
> with(numtheory):
Warning, the protected name order has been redefined and unprotected
> evalf(19/6);evalf(22/7);evalf(25/8);
3.16666667 3.142857143 3.125000000
> cfrac(Pi,3);
3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1 + ...}}}
> 3+1/(7+1/(15+1/(1)));
355
> evalf(%);
3.141592920
```

10. Check that $\sqrt{2 \sqrt{19549} + 286}$ is equal to $\sqrt{113} + \sqrt{173}$.

```maple
> sqrt(2*sqrt(19549)+286)-(sqrt(113)+sqrt(173));
0
> verify(sqrt(2*sqrt(19549)+286),sqrt(113)+sqrt(173));
true
> evalb(sqrt(2*sqrt(19549)+286)=sqrt(113)+sqrt(173));
true
> restart;
```

11. In Maple, transform $\frac{1}{\sqrt{3+1}}$ into an expression of the form $a + b\sqrt{3}$, with rational numbers $a$ and $b$.

```maple
> rationalize(1/(sqrt(3)+1));
-\frac{1}{2} + \frac{1}{2}\sqrt{3}
```

12. Let $\theta$ be a root of the polynomial $\theta^3 - \theta - 1$ and consider the extension of the field of rational numbers with $\theta$. So, we consider expressions of the form $a + b\theta + c\theta^2$, where $a$, $b$, and $c$ are rational numbers, and in calculations with these expressions we apply the identity $\theta^3 = \theta + 1$. Transform with Maple $\frac{1}{\theta^2+1}$ into an expression of the form $a + b\theta + c\theta^2$, where $a$, $b$, and $c$ are rational numbers.

```maple
> alias(theta=RootOf(x^3-x-1)):
> simplify(1/(theta^2+1));
\frac{4}{5} - \frac{2}{5} \theta^2 + \frac{1}{5} \theta
```

13. Show that Maple knows that the exponential power of a complex number can be written in terms of cosine and sine of the real and imaginary parts of that number. Also calculate $e^{(\frac{4\pi}{5})}$ in that form.

```maple
> convert(exp(I*z),trig);
\cos(z) + I \sin(z)
```
> assume(x,real):assume(y,real):
> evalc(exp(x*I*y));
>   \( e^{x^*} \cos(y^*) + I e^{x^*} \sin(y^*) \)
> evalc(exp(Pi*I/12));
>   \( \cos\left(\frac{1}{12} \pi\right) + I \sin\left(\frac{1}{12} \pi\right) \)
> convert(%,radical);
>   \( \frac{1}{4} \sqrt{2} (1 + \sqrt{3}) + \frac{1}{4} I \sqrt{2} (\sqrt{3} - 1) \)

14. Show with Maple that \( \tanh\left(\frac{z}{2}\right) = \frac{\sinh(x) + I \sin(y)}{\cosh(x) + \cos(y)} \), for any complex number \( z = x + y I \) with real \( x \) and \( y \).

> restart:
> z:=x+I*y;
>   \( z := x + I y \)
> evalc(tanh(z/2));
> \[
> \frac{\sinh\left(\frac{1}{2} x\right) \cosh\left(\frac{1}{2} x\right)}{\sinh\left(\frac{1}{2} x\right)^2 + \cos\left(\frac{1}{2} y\right)^2} + \frac{I \sin\left(\frac{1}{2} y\right) \cos\left(\frac{1}{2} y\right)}{\sinh\left(\frac{1}{2} x\right)^2 + \cos\left(\frac{1}{2} y\right)^2}
> \]
> combine(%);
> \[
> \frac{\sinh(x) + I \sin(y)}{\cosh(x) + \cos(y)}
> \]