MA214–02 Spring 2011
Quiz #6 - 10 Points
22 April 2011

NAME: Solutions

1. (4 points). Find the inverse Laplace transform of the function $G(s)$ given by

$$G(s) = \frac{s}{s^2 - 4s + 8}.$$

$s^2 - 4s + 8 = (s-2)^2 + 4$

$G(s) = \frac{s}{(s-2)^2+4} = \frac{s-2}{(s-2)^2+2^2} + \frac{2}{(s-2)^2+2^2}$

To use $(\mathcal{L}f)(s-b) \to e^{bt} f(t)$ everywhere $(\mathcal{L}f)(s)$ has an s there must now be $(s-b)$. In our case $b=2=a$.

ILT: • $\frac{s-2}{(s-2)^2+2^2}$. looks like $\frac{s}{s^2+2^2}$ with s replaced by $(s-2)$,

ILT $= e^{2t}\cos 2t$.

• $\frac{2}{(s-2)^2+2^2} \to e^{2t}\sin 2t$

$\boxed{(\mathcal{L}^{-1}G)(t) = e^{2t}[\cos 2t + \sin 2t]}$

2. (6 points). Find the unique solution to the initial value problem using the Laplace transform method:

$$y''(t) + 9y(t) = u_3(t), \quad y(0) = y'(0) = 0.$$

Take LT: $(s^2+9)(\mathcal{L}y)(s) = \frac{e^{-3s}}{s}$. Remember; $u_3(t) = u_3(t)\cdot 1$

$\Rightarrow (\mathcal{L}y)(s) = e^{-3s}\frac{1}{s(s^2+9)}$. Partial fraction: $\frac{1}{s(s^2+9)} = \frac{A}{s} + \frac{Bs+C}{s^2+9}$

(s^2+9 irreducible)

Cross Multiply: $1 = A(s^2+9) + (Bs+C)s = (A+B)s^2 + Cs + 9A$ so $C=0$

$A+B=0$ and $A=\frac{1}{9} \Rightarrow B = -\frac{1}{9}$.

$(\mathcal{L}y)(s) = \frac{1}{9}\left[e^{-3s}\frac{1}{s} - e^{-3s}\frac{s}{s^2+9}\right]$. Use $u_c(t)f(t-c)$ has LT $e^{-cs}(\mathcal{L}f)(s)$ with $c=3$.

$\downarrow$

$y(t) = \frac{1}{9}[u_3(t) - u_3(t)\cos[3(t-3)]]$. You can check this answer for $0<t<3$ - it is $y(t)=0$ and $t>3$;

$y(t) = \frac{1}{9}(1-\cos 3(t-3))$

Substitute into $y''+9y=1$ and you see this is the soln.

$\boxed{y(t) = \frac{1}{9}u_3(t)[1-\cos(3(t-3))]}$