MA533 Partial Differential Equations
Fall 2011
Problem Set 2
September 12, 2011
DUE: Wednesday, 21 September

(1) Let $U \subset \mathbb{R}^n$ have a smooth boundary (we only need C^1). Use Green's Theorem to prove that if u is harmonic in $U \subset \mathbb{R}^n$ and $u \in C^1(\overline{U})$, then for any $x \in U$, we have the representation

$$u(x) = \int_{\partial U} [u(y)\nu \cdot \nabla_y \Phi(x-y) - \Phi(x-y)\nu \cdot \nabla_y u(y)]\; dS(y),$$

where Φ is the fundamental solution for the Laplacian.

(2) Evans, pg. 85, #3. Prove a modified mean value formula for the solution to Poisson's PDE. HINT: use the formulas for $\phi'(r)$ involving the integral over the surface of a ball and the representation of $\phi'(r)$ involving the Laplacian on the top of page 26. The fundamental theorem of calculus and integration by parts will be useful. You will also need Theorem 3 in C.3.

(3) Use the Mean Value Theorem to prove that if u is a bounded harmonic function on $\mathbb{R}^n$, then u is a constant on $\mathbb{R}^n$. (There is a different proof on page 30 in Evans.)