MA676 Spring 2009
Homework Problem Set #6
Due: Friday, 17 April 2009
April 7, 2009

These problems are on the material in chapter 3 of Stein-Shakarchi. These problems are due Friday, 17 April 2009. Problem discussion Friday, 10 April at 4PM. (WZ means the problems are from Wheeden-Zygmund).

1. Let $f(x) = \chi_{[0,1]}(x)$ be the characteristic function on $[0,1]$ in R^1. Show that there exist constants $C, c > 0$ so that the maximal function of f satisfies:
$$\frac{C}{|x|} \geq f^*(x) \geq \frac{c}{|x|},$$
for large $|x|$. Conclude that $f^* \notin L^1(R)$. See S^2 exercise #4 on page 146 for the general case.

2. S^2, page 146, problem 6 on the one-sided maximal function in one dimension.

3. S^2, page 147, problem 11 on bounded variation.

4. If $\int_E f = 0$ for every measurable subset $E \subset R^d$, then $f = 0$ almost everywhere.

5. Show that the Cantor ternary function on $[0,1]$ is not absolutely continuous from the definition. This is an example of a continuous monotone increasing function that is not absolutely continuous.