Suppose that R and S are rings. An abelian group M is an (S, R)-*bimodule* if M is a left S-module, a right R-module and $s(mr) = (sm)r$ for all $s \in S$, $r \in R$, and $m \in M$.

- If $\phi\colon S \to R$ is a ring homomorphism let ${}_{\phi}R$ be R as an abelian group. Define a map
$$S \times {}_{\phi}R \times R \to {}_{\phi}R$$
by $(s, r, r') \mapsto \phi(s)rr'$. Show this makes ${}_{\phi}R$ an (S, R)-bimodule.

If M is a (S, R)-bimodule and N is an (R, T)-bimodule for rings S, R, T then $M \otimes_R N$ is an (S, T)-bimodule.

- Identify the bimodule ${}_{\phi}R \otimes_R M$. (That is - prove it is isomorphic to something else.)
- If $\phi\colon R' \to R$ is a ring homomorphism prove
$$M \otimes_R N \cong (M_{\phi}) \otimes_{R'} ({}_{\phi}N)$$