Assignment 12

1. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 6 \end{bmatrix}$.

   (a) Find the projection of $\vec{b}$ onto the column space of A.

   (b) Use the projection in (a) to find the least squares solution to $A\vec{x} = \vec{b}$.

   (c) Find the least squares solutions to $A\vec{x} = \vec{b}$ using the transpose of A.

2. Give the least squares solutions to the system

$$\begin{aligned} x - y &= 4 \\ x - y &= 6 \end{aligned}$$

3. For the data $(1,0)$, $(2,1)$, $(4,2)$, $(5,3)$ find the equation of best fit of the form

   (a) $ax + b$,

   (b) $ax^2 + bx$

4. Suppose radioactive substances A and B have decay constants of .02 and .07 respectively. If a mixture of these two substances at time $t = 0$ contains M_A grams of A and M_B grams of B, then a model for the total amount of y of the mixture present at times t is

$$y = M_A e^{-.02t} + M_B e^{-.07t}$$

   Suppose the initial amounts M_A and M_B are unknown, but a scientist is able to measure the total amount present at several times and records the following points (t, y): $(10, 21.34)$, $(11, 20.68)$, $(12, 20.05)$, $(14, 18.87)$, $(15, 18.30)$

   (a) What least squares problem do you need to solve to find M_A and M_B.

   (b) (Use technology!) Solve this least squares problem.