MA 322 - 09

Assignment 13

1. Is the set of all polynomials $\mathbf{p}(x)$ in $\mathbb{P}_n$ so that $\mathbf{p}(0) = 0$ a subspace of $\mathbb{P}_n$? *Justify!*
2. Fix a 3×2 matrix F. If H is the set of all matrices A in $M_{2\times 4}$ so that $FA = 0$, is H a subspace of $M_{2\times 4}$? *Justify!*
3. Let $T\colon \mathbb{P}_2 \to \mathbb{M}_{2\times 2}$ be the transformation $T(p) = \begin{bmatrix} p(0) & p(-1) \\ p(1) & p(2) \end{bmatrix}$.
   (a) Show T is linear.
   (b) What is the kernel of T?
   (c) What is the range of T?

   Let $S\colon \mathbb{P}_3 \to \mathbb{M}_{2\times 2}$ be the transformation $S(p) = \begin{bmatrix} p(0) & p(-1) \\ p(1) & p(2) \end{bmatrix}$. (The only difference between S and T is the type of input.)
   (d) What is the kernel of S?
   (e) What is the range of S?
4. Let C be the vector space of all continuous real valued functions of a single real variable. There is a linear transformation $D\colon C \to C$ defined by $D(f)$ is the derivative of f. What is the kernel of D?
5. Let $\vec{p}_1(t) = 1 + t$, $\vec{p}_2(t) = 1 - t$, $\vec{p}_3(t) = 2$.
   (a) Are $\{\vec{p}_1(t), \vec{p}_2(t), \vec{p}_3(t)\}$ a linearly independent set?
   (b) Find a basis for the span of $\{\vec{p}_1(t), \vec{p}_2(t), \vec{p}_3(t)\}$.
   (c) What is the coordinate vector of $4x + 3$ relative to the basis you chose in (b)?
6. Is $\{t, \sin(t), \cos(t)\}$ a linearly independent set of vectors? (Hint: functions are the same only when the values for all choices of t are the same.)