MA 322 - 09

Assignment 3

1. (a) Compute the span of $\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$.

   (b) Give a geometric description of this span.

2. (a) Compute the span of $\begin{bmatrix} 4 \\ 2 \\ 10 \end{bmatrix}$ and $\begin{bmatrix} 6 \\ 3 \\ 15 \end{bmatrix}$.

   (b) Give a geometric description of this span.

3. How many rows of $A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 4 & 4 & 4 & 0 \\ -1 & 0 & -3 & 1 \\ 2 & 2 & 2 & -2 \end{bmatrix}$ have a pivot position? Does the equation $A\vec{x} = \vec{b}$ have a solution for each $\vec{b}$ in $\mathbb{R}^4$?

4. Let A be a 3×2 matrix (so three rows and two columns). Explain why the equation $A\vec{x} = \vec{b}$ cannot be solved for every $\vec{b}$ in $\mathbb{R}^3$. What about A a 4×3 matrix?

5. If A is a 3×3 matrix and $\vec{v}_1, \vec{v}_2$, $\vec{y}_1, \vec{y}_2$ are vectors so that $A\vec{y}_1 = \vec{v}_1$ and $A\vec{y}_2 = \vec{v}_2$ find a vector $\vec{w}$ so that $A\vec{w} = \vec{v}_1 + 3\vec{v}_2$.

6. Give a geometric comparison of the solutions to

$$\begin{array}{rrrr} x_1 & +3x_2 & -5x_3 & = 4 \\ x_1 & +4x_2 & -8x_3 & = 7 \\ -3x_1 & -7x_2 & +9x_3 & = -6 \end{array} \quad \text{and} \quad \begin{array}{rrrr} x_1 & +3x_2 & -5x_3 & = 0 \\ x_1 & +4x_2 & -8x_3 & = 0 \\ -3x_1 & -7x_2 & +9x_3 & = 0 \end{array}$$

7. Suppose $A\vec{x} = \vec{b}$ has a solution. Explain why the solution is unique exactly when $A\vec{x} = \vec{0}$ has only the trivial solution.