MA 322 - 09

Assignment 4

1. Describe the possible echelon forms for matrices with the following properties:

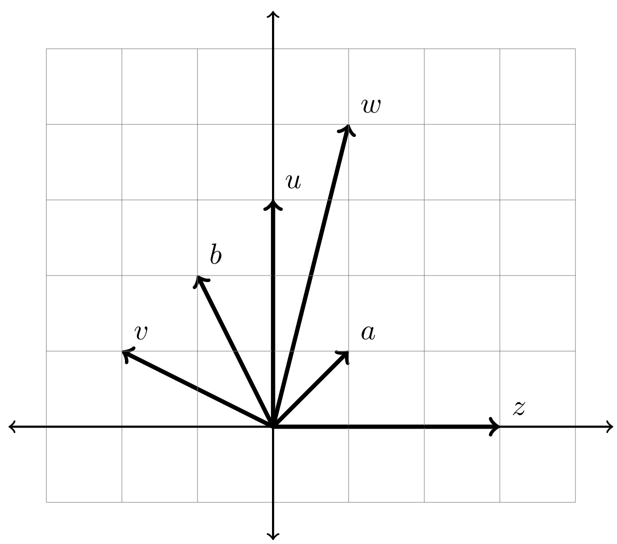
   (a) A is a 2×2 matrix with linearly dependent columns.

   (b) A is a 4×3 matrix, $A = [\vec{a}_1\, \vec{a}_2\, \vec{a}_3]$, such that $\{\vec{a}_1, \vec{a}_2\}$ is linearly independent and $\vec{a}_3$ is not in the span of $\vec{a}_1$ and $\vec{a}_2$.

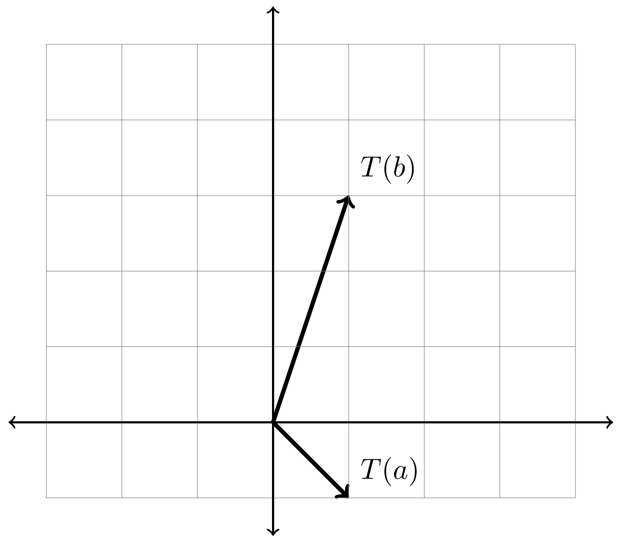
2. Suppose an $m \times n$ matrix A has n pivot columns. Explain why the equation $A\vec{x} = \vec{b}$ can have at most one solution for each $\vec{b}$.

3. Show that the transformation T defined by $T(x_1, x_2) = (2|x_2|, 3x_1 - x_2)$ is not linear.

4. This figure shows the vectors $\vec{a}$, $\vec{b}$, $\vec{u}$, $\vec{v}$, $\vec{w}$, and $\vec{z}$.



   If $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and the images of $\vec{a}$ and $\vec{b}$ are shown below, draw the images of $\vec{u}$, $\vec{v}$, $\vec{w}$, and $\vec{z}$.



5. Let $\vec{u}$ and $\vec{v}$ be vectors in $\mathbb{R}^2$. It can be shown that the set P of all points in the parallelogram determined by $\vec{u}$ and $\vec{v}$ has the form $a\vec{u} + b\vec{v}$, for $0 \leq a \leq 1$, $0 \leq b \leq 1$. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Explain why the image of a point in P under the transformation T lines in the parallelogram determined by $T(\vec{u})$ and $T(\vec{v})$.

6. (a) Give the matrix for the transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ that first reflects points through the x-axis and then reflects through the line $y = x$.

   (b) How else can you describe this transformation?