MA 322 - 09

# Assignment 9

1. Diagonalize the matrix
$$\begin{bmatrix} 0 & -4 & -6 \\ -1 & 0 & -3 \\ 1 & 2 & 5 \end{bmatrix}$$
if possible. (Hint: 1 and 2 are eigenvalues.)

2. Construct a nondiagonal $2 \times 2$ matrix that is diagonalizable but not invertible.

3. Let $A = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}$, $\vec{b}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $\vec{b}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, and $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$. Find matrix $A$ so that $[T(\vec{v})]_{\mathcal{B}} = A[\vec{v}]_{\mathcal{B}}$.

4. Let $T(\vec{v}) = \begin{bmatrix} 5 & -3 \\ -7 & 1 \end{bmatrix} \vec{v}$. Find a basis $\mathcal{B}$ for $\mathbb{R}^2$ so that the matrix of $T$ is diagonal with respect to $\mathcal{B}$.

5. Find the eigenvalues and eigenvectors of $\begin{bmatrix} 5 & -5 \\ 1 & 1 \end{bmatrix}$.

6. Suppose $A$ is an $n \times n$ real matrix and $A\vec{v} = \lambda\vec{v}$ for some complex number $\lambda = (a + ib)$.
   (a) Write $A(\mathrm{Re}(\vec{v}) + i\mathrm{Im}(\vec{v}))$ using $a$, $b$, $\mathrm{Re}(\vec{v})$, and $\mathrm{Im}(\vec{v})$.
   (b) Show $A(\mathrm{Re}(\vec{v})) = a\mathrm{Re}(\vec{v}) + b\mathrm{Im}(\vec{v})$ and $A(\mathrm{Im}(\vec{v})) = -b\mathrm{Re}(\vec{v}) + a\mathrm{Im}(\vec{v})$.