

3 The Mathematics of Sharing

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3.4 The Lone-Chooser Method

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Divider-Chooser Method

The **divider-chooser method** (also called the *you cut–I choose method*) can be used when the fair-division game involves two players and a *continuous* set S

As this name suggests, one player, called the *divider*, divides S into two shares, and the second player, called the *chooser*, picks the share he or she wants, leaving the other share to the divider.

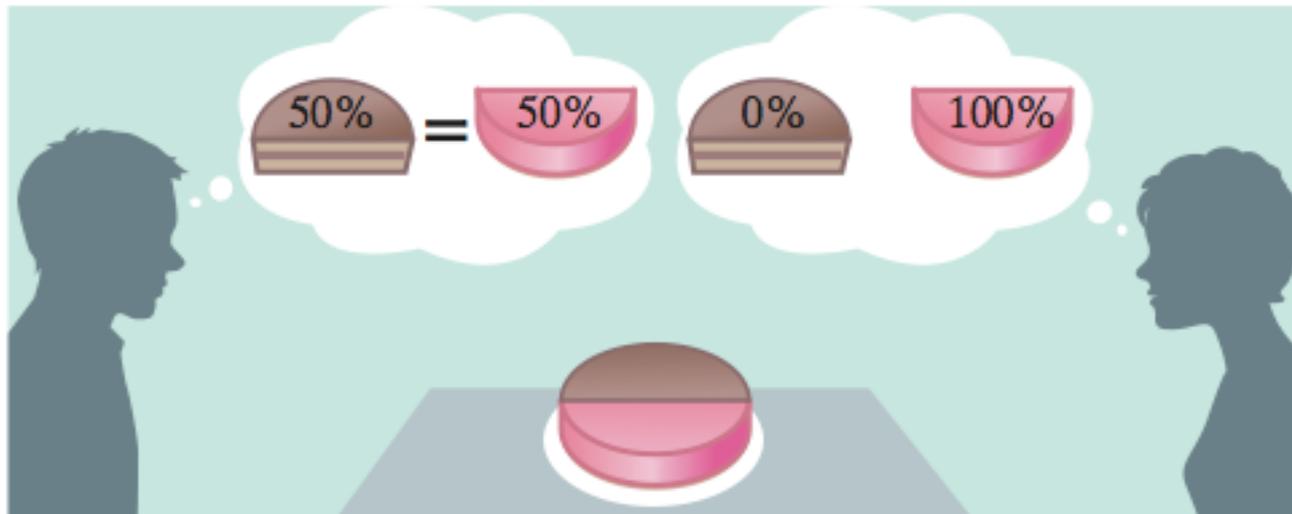
Divider-Chooser Method

This method guarantees that divider and chooser will each get a fair share (with two players, this means a share worth 50% or more of the total value of S).

Not knowing the chooser's likes and dislikes (privacy assumption), the divider can only guarantee himself a 50% share by dividing S into two halves of equal value (rationality assumption); the chooser is guaranteed a 50% or better share by choosing the piece he or she likes best.

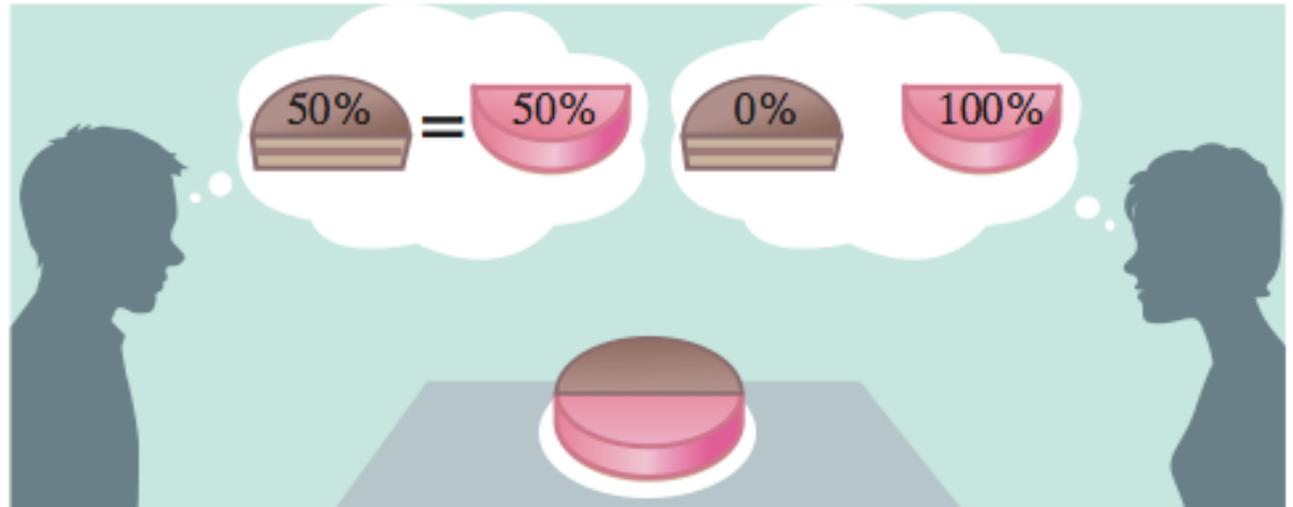
Damian and Cleo Divide a Cheesecake

On their first date, Damian and Cleo go to the county fair. They buy jointly a raffle ticket and win a half chocolate–half strawberry cheesecake. Damian likes chocolate and strawberry equally well, so in his eyes the chocolate and strawberry halves are equal in value.



Damian and Cleo Divide a Cheesecake

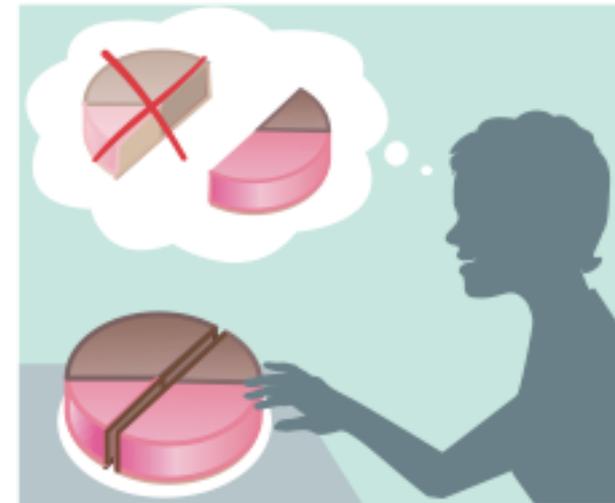
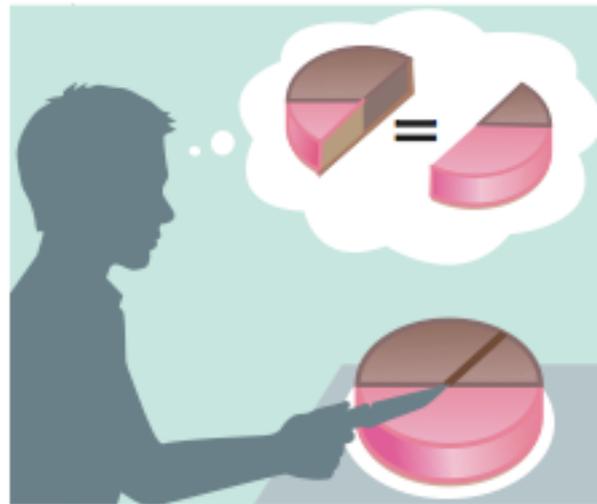
However, Cleo hates chocolate so the chocolate part of the cake is worth 0% of the whole cake, and the strawberry part is worth 100% of the whole cake (as far as Cleo is concerned).



To ensure a fair division, we assume neither of them knows anything about the other's likes and dislikes.

Damian and Cleo Divide a Cheesecake

Damian volunteers to go first (the divider). According to Damien's value system, any physical half of the cake is a fair share, so he cuts the cake into two halves, ignoring the amount of strawberry / chocolate in either half. It is now Cleo's turn to choose, and her choice is obvious: she will pick the piece having the larger strawberry part.



Damian and Cleo Divide a Cheesecake

Final outcome:

Damian gets a piece that is worth exactly half of the cake (According to Damian's value system)

Cleo ends up with a much sweeter deal—a piece that in her own eyes is worth about two-thirds of the cake. (According to Cleo's value system)

This is a fair division of the cake—both players get pieces worth 50% or more (according to their respective value systems)

Better to be the Chooser

This example illustrates that it is better to be the chooser than the divider.

The divider is guaranteed a share worth exactly 50% of the total value of S ,

The chooser could end up with a share worth more than 50%.

(If the players each had the same value system they would each end up with exactly 50%. The differences between their value systems is what allows the chooser to (potentially) end up with more than 50%)