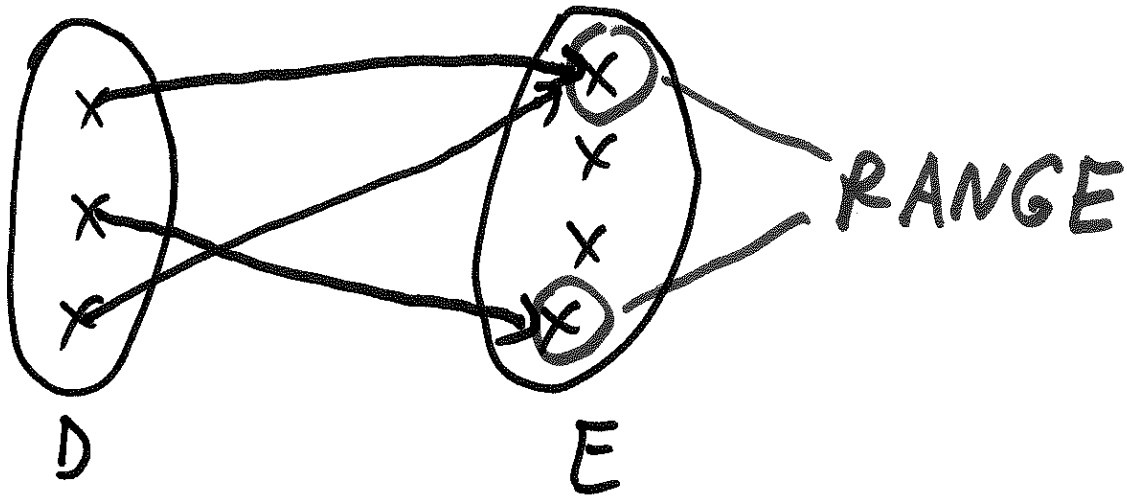
LECTURE 1

FUNCTIONS



D = DOMAIN

E = CO-DOMAIN

f = RULE

EACH x IN D IS ASSIGNED EXACTLY ONE y IN E

RANGE OF f = ALL THE POSSIBLE VALUES f TAKES

x = INDEPENDENT VARIABLE

$f(x) = y$ = DEPENDENT VARIABLE

$(f+g)(x) = f(x)+g(x)$

$(f-g)(x) = f(x)-g(x)$

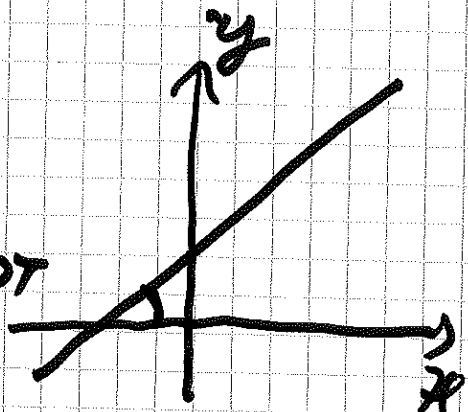
$(f\cdot g)(x) = f(x)\,g(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ IF $g(x)\neq 0$

EX: LINEAR FUNCTIONS

$f(x) = mx+n$

m: SLOPE, n: y-INTERCEPT



POWERS $x\cdot x = x^2$

POLYNOMIALS $a_n x^n + a_{n-1}x^{n-1} + \dots + a_1 x + a_0$

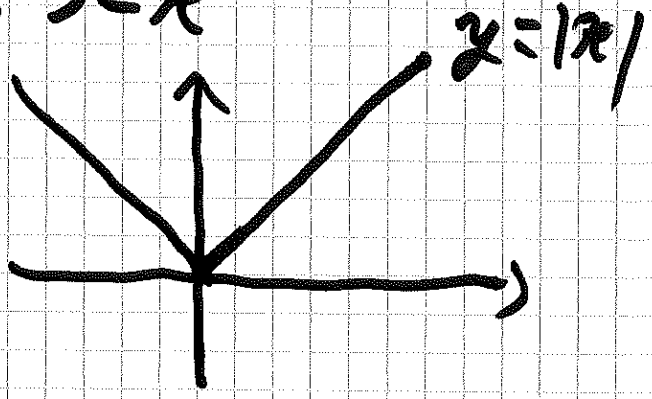
RATIONAL FUNCTIONS $\frac{P(x)}{Q(x)}$

EX: $f(x) = \frac{3x+5}{x^2-4}$ DOMAIN: $x \neq \pm 2$

PIECEWISE FUNCTIONS

EX: $f(x) = \begin{cases} 5x+7, & x \le 1 \\ \frac{x}{5x^2+3}, & 1 < x \le 5 \\ 2x-5, & 5 < x \end{cases}$

$|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0 \end{cases}$



INVERSE OF A FUNCTION

$f: D \to f(D)$ = RANGE

f IS <u>ONE-TO-ONE</u> IF FOR ALL $x_1 \neq x_2$, $f(x_1) \neq f(x_2)$

IF f IS ONE-TO-ONE:

$y = f(x)$, SOLVE FOR x

$x = g(y)$ NOTATION: $g = f^{-1}$

$f: (0, +\infty) \to (0, +\infty)$

$f(x) = x^2$

$x^2 = y$, $x = \sqrt{y}$

$f^{-1}(x) = \sqrt{x}$

D IS A SUBSET OF REAL NUMBERS

INTERVALS [ ] [ ) ( ] ( )

$[a,b]$ $a \le x \le b$

(a,b) $a < x < b$

BASIC FUNCTIONS:

I) $f(x) = 1$ FOR ALL x

DOMAIN $(-\infty, +\infty)$

RANGE $y = 1$

II) $f(x) = x$

DOMAIN $(-\infty, +\infty)$

RANGE $(-\infty, +\infty)$

III) CONSTRUCT NEW FUNCTIONS

$f, g : D \to E$, C CONSTANT

$(Cf)(x) = C f(x)$

$f(x) = x^2 + 4$

$D = [0, +\infty)$, $f(D) = [4, +\infty)$

SOLVE: $y = x^2 + 4$

$y - 4 = x^2$ $\quad x = \sqrt{y-4}$

$f^{-1}(x) = \sqrt{x-4}$

COMPOSITION OF FUNCTIONS

$(f \circ g)(x) = f(g(x))$

OBS 1: g IS APPLIED FIRST

OBS 2: $f \circ g \neq g \circ f$

EX: $f(x) = \sqrt{x}$, $g(x) = x^3 - 8$

$(f \circ g)(x) = \sqrt{x^3 - 8}$

$(g \circ f)(x) = (\sqrt{x})^3 - 8$

OBS 3: $(f \circ f^{-1})(x) = x$

$(f^{-1} \circ f)(x) = x$