IVT $f:[a,b]\to\mathbb{R}$ CONTINUOUS, THEN IF N IS BETWEEN $f(a)$ AND $f(b)$ THEN THERE IS $a\le c\le b$ SO THAT $f(c)=N$

EX: SOLVE $x^5+x-1=0$

$f(x)=x^5+x-1$ CONTINUOUS EVERYWHERE

$f(0)=-1$ THUS THERE IS $0<c<1$

$f(1)=1$ SO THAT $f(c)=0$

$f(\frac{1}{2})=(\frac{1}{2})^5+\frac{1}{2}-1<0$ THUS $\frac{1}{2}<c<1$

$f(1)>0$

EX: $x=\sqrt{1+\sin^2 x}$ HAS A SOLUTION $0<c<\pi$

$f(x)=x-\sqrt{1+\sin^2 x}$

$f(0)=0-1=-1<0$

$f(\pi)=\pi-\sqrt{1+\sin^2\pi}=\pi-1\approx 2.14>0$

EX: $H : [-1,1] \to \mathbb{R}$

$$H(x) = \begin{cases} 0, & -1 \leq x < 0 \\ 1, & 0 \leq x \leq 1 \end{cases}$$

$H(-1) = 0, \quad H(1) = 1$

$H(c) \neq \frac{1}{2}$ FOR ALL $-1 \leq c \leq 1$

LIMITS AT INFINITY

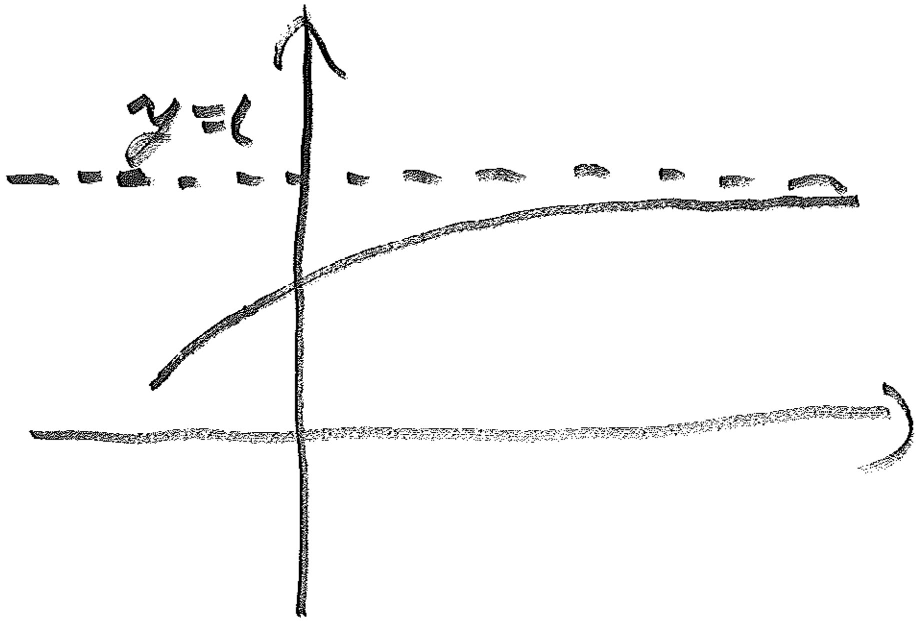
"DEF": $\lim_{x \to \infty} f(x) = L$ IF:

i) f IS WELL-DEFFINED ON $(M, +\infty)$ FOR SOME M

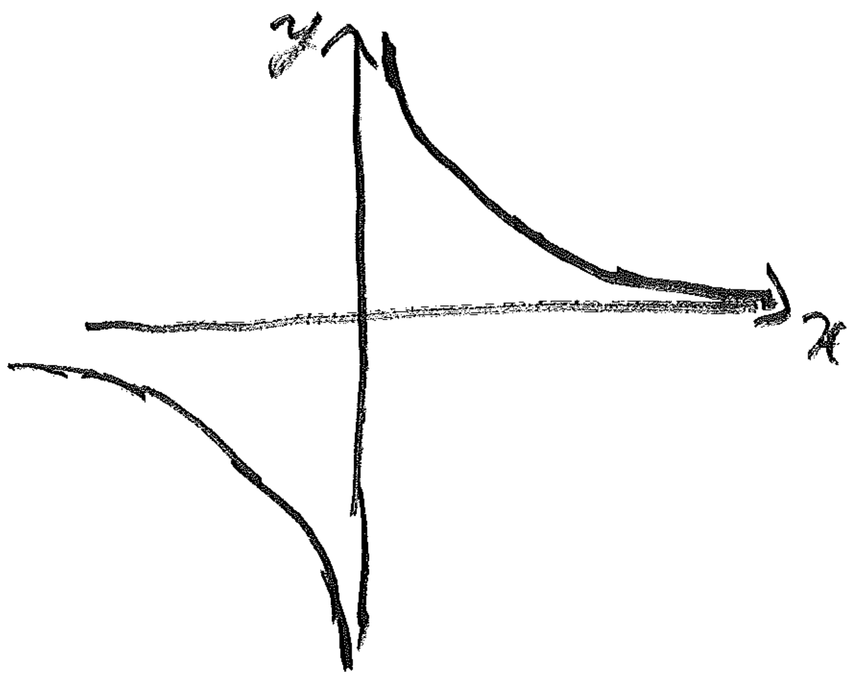
ii) THE VALUES OF f GET "ARBITRARILY CLOSE" TO L AS x IS "SUFFICIENTLY LARGE"

SIMILARLY $\lim_{x \to -\infty} f(x) = L$

DEF: $y = L$ IS A HORIZONTAL ASYMPTOTE IF $\lim_{x \to +\infty} f(x) = L$ (OR $\lim_{x \to -\infty} f(x) = L$)



$\underline{EX}: f(x) = \frac{1}{x}$



$x = 0$ VERTICAL

$y = 0$ HORIZONTAL

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to -\infty} \frac{1}{x} = 0$$

$$\lim_{x \to \infty} \frac{1}{x^a} = 0, \quad a > 0$$

$$\lim_{x \to -\infty} \frac{1}{x^a} = 0, \quad a > 0$$

(WHEN DEFINED)

LIMIT LAWS STILL APPLY

"DEF" $\lim_{x \to \infty} f(x) = \infty$ MEANS

THAT $f(x)$ GETS "ARBITRARILY LARGE"

IF x IS "SUFFICIENTLY LARGE"

SIMILARLY $\lim_{x \to \infty} f(x) = -\infty$

$$\lim_{x \to -\infty} f(x) = \pm\infty$$

<u>EX</u>: $\lim_{x \to \infty} x = +\infty$

$$\lim_{x \to \infty} x^a = \begin{cases} +\infty & \text{IF } a > 0 \\ 0 & \text{IF } a < 0 \\ 1 & \text{IF } a = 0 \end{cases}$$

EX: $\lim_{x \to \infty} (2x+5) = \lim_{x \to \infty} 2x + \lim_{x \to \infty} 5$

$$= 2 \lim_{x \to \infty} x + 5 \text{ "="} \, 2 \cdot \infty + 5 = +\infty$$

" $\infty + \infty \stackrel{"}{=} \infty$ " " $\infty - 1{,}000{,}000 \stackrel{"}{=} \infty$ "

$$\text{"}\infty - \infty = ??\text{"}$$

$$\text{"}\frac{\infty}{\infty} = ??\text{"}$$

$$\lim_{x\to\infty} \frac{x^2}{x} = \lim_{x\to\infty} x = +\infty$$

$$\lim_{x\to\infty} \frac{x}{x^2} = \lim_{x\to\infty} \frac{1}{x} = 0$$

$$\text{"}\frac{0}{\infty} = 0\text{"}$$

$$\text{"}\frac{\infty}{0} = \pm\infty\text{"}$$

$$\text{"}0 \cdot \infty \doteq ??\text{"}$$

$$\lim_{x\to\infty} \frac{1}{x} \cdot x = 1$$

$\underline{\text{Ex}}$: $$\lim_{x\to+\infty} \frac{2x^2+2}{3x^2+5x+1}$$

$$\lim_{x\to+\infty} 2x^2+2 = +\infty \qquad \text{"}\frac{\infty}{\infty}\text{"}$$

$$\lim_{x\to+\infty} 3x^2+5x+1 = +\infty$$

$$2x^2+2 \approx 2x^2 \qquad \lim_{x\to+\infty}\frac{2x^2}{3x^2}=\frac{2}{3}$$

$$3x^2+5x+1 \approx 3x^2$$

$$\lim_{x\to+\infty}\frac{2x^2+2}{3x^2+5x+1}=\lim_{x\to+\infty}\frac{\cancel{x^2}\left(2+\frac{2}{x^2}\right)}{\cancel{x^2}\left(3+\frac{5}{x}+\frac{1}{x^2}\right)}$$

$$=\lim_{x\to+\infty}\frac{2+\frac{2}{x^2}}{3+\frac{5}{x}+\frac{1}{x^2}}=\frac{\lim_{x\to+\infty}\left(2+\frac{2}{x^2}\right)}{\lim_{x\to+\infty}\left(3+\frac{5}{x}+\frac{1}{x^2}\right)}=\frac{2}{3}$$

IN GENERAL

$$\lim_{x\to\infty}\frac{a_1x^n+a_2x^{n-1}+\dots}{b_1x^m+b_2x^{m-1}+\dots}=\begin{cases}\pm\infty, & n>m\\ 0, & n<m\\ \frac{a_1}{b_1}, & n=m\end{cases}$$

$$\lim_{x\to+\infty}\sin x \quad \text{DNE}$$

$$\lim_{x \to \infty} \sqrt{x+100} - \sqrt{x}$$

$$\text{"}\infty - \infty\text{"} \quad ??$$

$$\sqrt{x+100} - \sqrt{x} = \frac{(\sqrt{x+100} - \sqrt{x})(\sqrt{x+100} + \sqrt{x})}{\sqrt{x+100} + \sqrt{x}}$$

$$= \frac{\cancel{x} + 100 - \cancel{x}}{\sqrt{x+100} + \sqrt{x}} = \frac{100}{\sqrt{x+100} + \sqrt{x}}$$

$$\lim_{x \to \infty} (\dots) = \frac{100}{\infty} = 0$$