LAST TIME: LIMITS AT INFINITY

$$\lim_{x \to \infty} f(x)$$

SUBSTITUTION: $x = \frac{1}{t}$ (OR $t = \frac{1}{x}$) $t > 0$

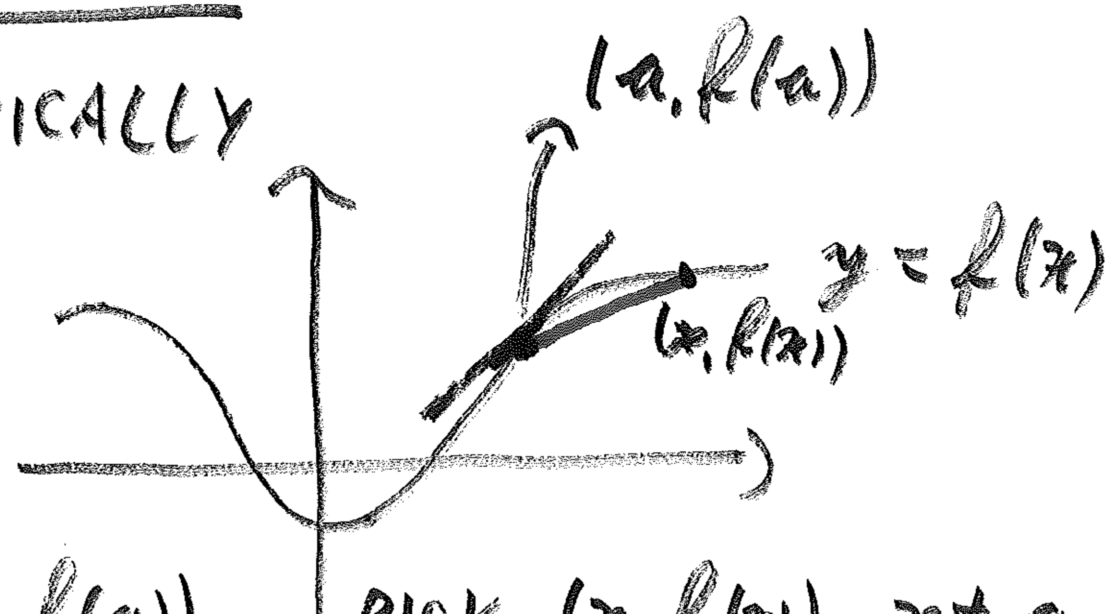
THIS BECOMES $\lim_{t \to 0^+} f\left(\frac{1}{t}\right)$

EX 1: $\lim_{x \to \infty} \sin\left(\frac{1}{x}\right) = \lim_{t \to 0^+} \sin(t) = \sin(0) = 0$

$$\lim_{x \to \infty} e^{-\frac{1}{x}} = \lim_{t \to 0} e^{-t} = e^{-0} = 1$$

DERIVATIVES

GEOMETRICALLY



FIX $(a, f(a))$, PICK $(x, f(x))$, $x \neq a$

$m =$ SLOPE OF LINE $= \dfrac{f(x)-f(a)}{x-a}$

LET $x \to a$

$$m = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

OR

$x-a=h, \quad x=a+h$

$$m = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

ALGEBRAICALLY

THE DERIVATIVE OF f AT a, CALLED $f'(a)$, IS $f'(a) = \displaystyle\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

(PROVIDED LIMIT EXISTS, FINITE)

<u>EX 1</u>: $f(x) = 2x+5$, $f'(2) = ?$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h)-f(2)}{h} = \cancel{\lim_{h \to 0} \frac{2(2+h)}{}}$$

$$= \lim_{h \to 0} \frac{2(2+h)+\cancel{5}-(2\cdot 2+\cancel{5})}{h} = \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}} = 2$$

EX 2: $f(x)=x^2-2x$, $f'(2)=?$

$$f'(2)=\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$=\lim_{h\to 0}\frac{(2+h)^2-2(2+h)-(2^2-2\cdot 2)}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{2^2}+2\cdot 2\cdot h+h^2-\cancel{2\cdot 2}-2h-\cancel{2^2}+\cancel{2\cdot 2}}{h}$$

$$=\lim_{h\to 0}\frac{h^2+2h}{h}=\lim_{h\to 0}\frac{\cancel{h}(h+2)}{\cancel{h}}$$

$$=\lim_{h\to 0}(h+2)=2$$

EX 3: $f(x)=\sqrt{x+2}$, $f'(2)=?$

$$f'(2)=\lim_{h\to 0}\frac{\sqrt{4+h}-\sqrt{4+0}}{h}=$$

$$=\lim_{h\to 0}\frac{\left(\sqrt{4+h}-\sqrt{4}\right)\left(\sqrt{4+h}+\sqrt{4}\right)}{h\left(\sqrt{4+h}+\sqrt{4}\right)}$$

$$=\lim_{h\to 0}\frac{\cancel{4}+h-\cancel{4}}{h\left(\sqrt{4+h}+2\right)}=\lim_{h\to 0}\frac{1}{\sqrt{4+h}+2}=\frac{1}{4}$$

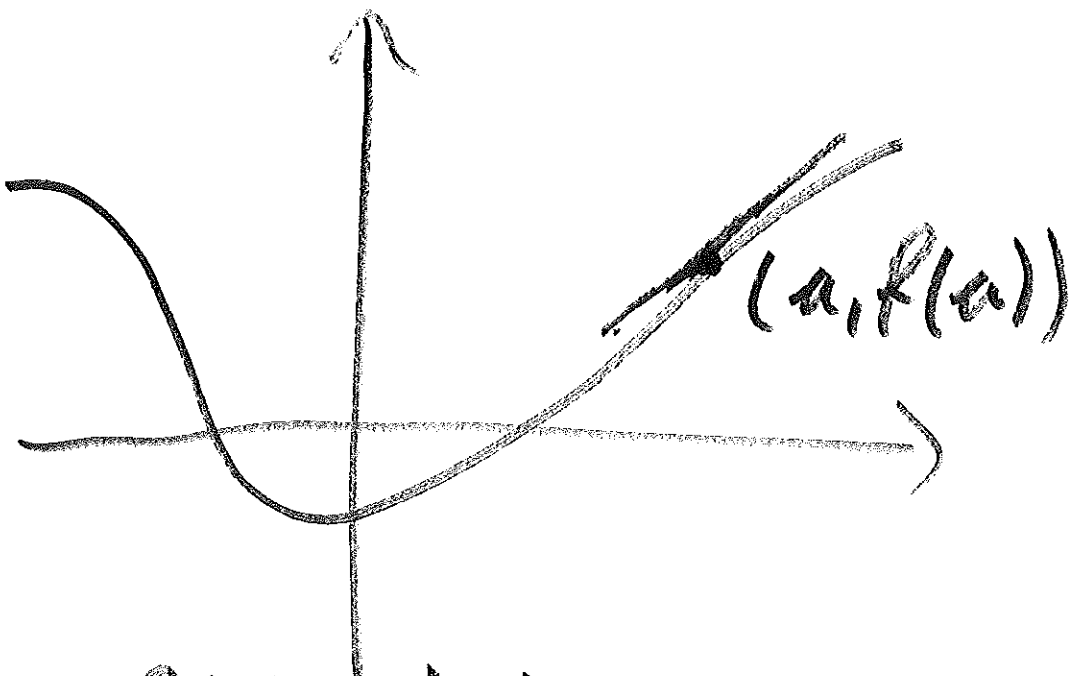
EX 4: $f(x)=\frac{1}{x}$, $f'(2)=?$

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{2-(2+h)}{2(2+h)}}{h}=\lim_{h\to 0}\frac{-h}{h\cdot 2(2+h)}$$

$$=\lim_{h\to 0}\frac{-1}{2(2+h)}=-\frac{1}{4}$$

EX 5: TANGENT LINE AT $(a, f(a))$



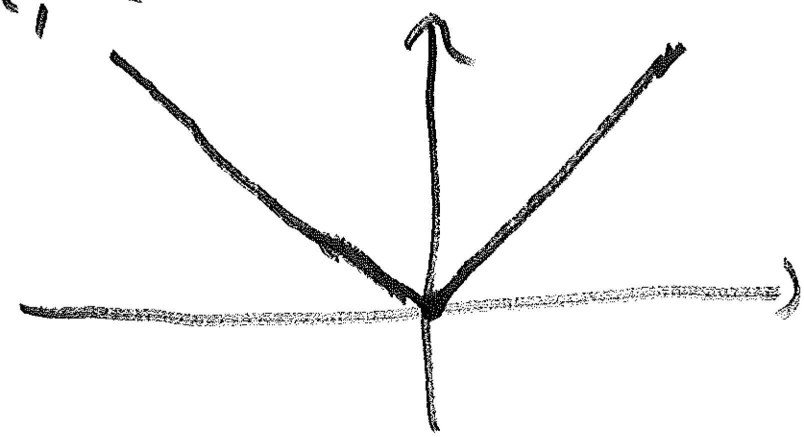
$$y-f(a)=f'(a)(x-a)$$

FIND TANGENT LINE TO THE GRAPH OF $f(x)=\frac{1}{x}$ AT $(2, \frac{1}{2})$.

$$y - \frac{1}{2} = -\frac{1}{4}(x-2)$$

EX 6: $f(x) = |x| = \{$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



$f'(0) = ?$

$$\lim_{h \to 0} \frac{f(h) - f(0)}{h} \text{ DNE}$$

$$\left.\begin{aligned} \lim_{h \to 0^+} \frac{f(h) - f(0)}{h} &= \lim_{h \to 0^+} \frac{h - 0}{h} = 1 \\ \lim_{h \to 0^+} \frac{f(h) - f(0)}{h} &= \lim_{h \to 0^-} \frac{-h - 0}{h} = -1 \end{aligned}\right\} \neq$$

EX 7: $f(x) = |x|^3 = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases}$

CHECK: $f'(0) = 0$

Ex 8 : $H(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$

$H'(0) = ?$ DNE

$$\lim_{h \to 0^+} \frac{H(h) - H(0)}{h} = \lim_{h \to 0^+} \frac{1-1}{h} = \lim_{h \to 0^+} 0 = 0$$

$$\lim_{h \to 0^-} \frac{H(h) - H(0)}{h} = \lim_{h \to 0^-} \frac{0-1}{h} = +\infty$$

Ex 9 : $f(t)$ = POSITION AT TIME t

THEN $f'(a)$ = INSTANTENOUS VELOCITY AT TIME a