DERIVATIVES OF POLYNOMIALS/EXP.

1) $f(x) = C$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{C - C}{h} = \lim_{h \to 0} \frac{0}{h}$$

$$= 0$$

THEN $f'(x) = 0$

2) $f(x) = x$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{x + h - x}{h} = \lim_{h \to 0} \frac{h}{h}$$

$$= 1$$

THEN $f'(x) = 1$

3) $f(x) = x^2$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$$= \lim_{h \to 0} (2x + h) = 2x$$

THEN $f'(x) = 2x$

$f(x) = x^3$

$$f(x+h) - f(x) = (x+h)^3 - x^3 = 3x^2h + 3xh^2 + h^3$$

$$(x+h)^3 = (x+h)(x+h)(x+h) = x^3 + 3x^2h + 3xh^2 + h^3$$

$$\lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h\to 0} \frac{h(3x^2 + 3xh + h^2)}{h}$$

$$= \lim_{h\to 0}(3x^2 + 3xh + h^2) = 3x^2$$

$$f'(x) = 3x^2$$

$f(x) = x^n$

$$(x+h)^n = \underbrace{(x+h)(x+h)\dots(x+h)}_{n} = x^n + nx^{n-1}h + h^2(\dots)$$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\cancel{x^n} + nx^{n-1}h + h^2(\dots) - \cancel{x^n}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(nx^{n-1} + h(\dots))}{\cancel{h}} = \lim_{h\to 0}(nx^{n-1} + h(\dots))$$

$$= nx^{n-1}$$

$\frac{d}{dx}(x^n) = nx^{n-1}$ FOR ALL POSITIVE INTEGERS n

ACTUALLY: $\frac{d}{dx}(x^r) = rx^{r-1}$ FOR ALL REAL NUMBERS r

EX: $\frac{d}{dx}(x^{-\frac{7}{2}}) = -\frac{7}{2} \cdot x^{-\frac{7}{2}-1} = -\frac{7}{2}x^{-\frac{9}{2}}$

DERIVATIVE LAWS

C CONSTANT, f, g FUNCTIONS

$(Cf)' = Cf'$

$(f+g)' = f' + g'$

$(f-g)' = f' - g'$

$$(Cf)'(x) = \lim_{h \to 0} \frac{Cf(x+h) - Cf(x)}{h}$$

$$= \lim_{h \to 0} C \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} C \cdot \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= C f'(x)$$

$\underline{\text{EX}}$: $f(x) = 7x^2$

$$f'(x) = (7x^2)' = 7(x^2)' = 7 \cdot 2x = 14x$$

$\underline{\text{EX}}$: $f(x) = 2x^5 + 7x^3 - x - 1$

$$f'(x) = (2x^5)' + (7x^3)' - (x)' - (1)'$$
$$= 2(x^5)' + 7(x^3)' - (x)' - (1)'$$
$$= 10x^4 + 21x^2 - 1$$

$$P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

$$P_n'(x) = n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \dots + a_1$$

EXPONENTIAL FUNCTIONS

$$f(x) = b^x$$

$$f'(x) = \lim_{h \to 0} \frac{b^{x+h} - b^x}{h} = \lim_{h \to 0} \frac{b^x b^h - b^x}{h}$$
$$= \lim_{h \to 0} b^x \cdot \frac{b^h - 1}{h} = \lim_{h \to 0} b^x \cdot \lim_{h \to 0} \frac{b^h - 1}{h}$$
$$= b^x \cdot f'(0)$$

$\underline{Q}$: WHAT IS $f'(0)$? DEPENDS ON b

PICK b SO THAT $\lim\limits_{h\to 0}\frac{b^h-1}{h}=1$

CORRECT CHOICE: $b=e$

$$\lim_{h\to 0}\frac{e^h-1}{h}=1$$

THEN: $\frac{d}{dx}(e^x)=e^x\cdot\frac{d}{dx}e^x\Big|_{x=0}=e^x$

FIND y SO THAT $\frac{dy}{dx}=y$

$$\frac{d}{dx}(5e^x)=5\frac{d}{dx}e^x=5e^x$$

$\frac{d}{dx}(Ce^x)=Ce^x$ FOR ANY CONSTANT C

WHAT ABOUT $f\cdot g$?

$$(f\cdot g)' \cancel{\stackrel{?}{=}} f'\cdot g'$$

$$(x^2)'=2x$$

$$(x\cdot x)' \qquad x'\cdot x'=1$$

Ex: $\left(x^2(x^2-4)\right)' = (x^4-4x^2)'$

$= (x^4)' - (4x^2)' = 4x^3 - 8x$

Ex: $\left((x^2+4)^{1000}\right)' = ??$