LECTURE 2

LAST TIME: FUNCTIONS

INVERSE

COMPOSITION

$f: D \to (-\infty, +\infty)$

$g: E \to (-\infty, +\infty)$

$(f \circ g)(x) = f(g(x))$

NEED: $g(E) \subset D$

ALL ELEMENTS IN THE RANGE OF g MUST BELONG TO D

<u>EX</u>: $h(x) = \sqrt[4]{3-\sqrt{x}}$

$h = f \circ g$

$f(x) = \sqrt[4]{x}$

$g(x) = 3 - \sqrt{x} : [0, +\infty)$

$g([0, +\infty)) = (-\infty, 3]$

NEED: $3 - \sqrt{x} \geq 0$

$\sqrt{x} \leq 3$

$0 \leq x \leq 9$

POWER FUNCTIONS $n \geq 0$ INTEGER

m INTEGER

$x^n = x \cdot \ldots \cdot x$

$x^{-n} = \frac{1}{x^n}$, $x \neq 0$

$x^{\frac{1}{n}} = \sqrt[n]{x}$

$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(x^m\right)^{\frac{1}{n}}$

IN GENERAL: DEFINE x^r FOR ALL REAL NUMBERS r

DOMAIN: $[0, +\infty)$

$x^0 = 1$

<u>EXPONENTIAL FUNCTION</u>

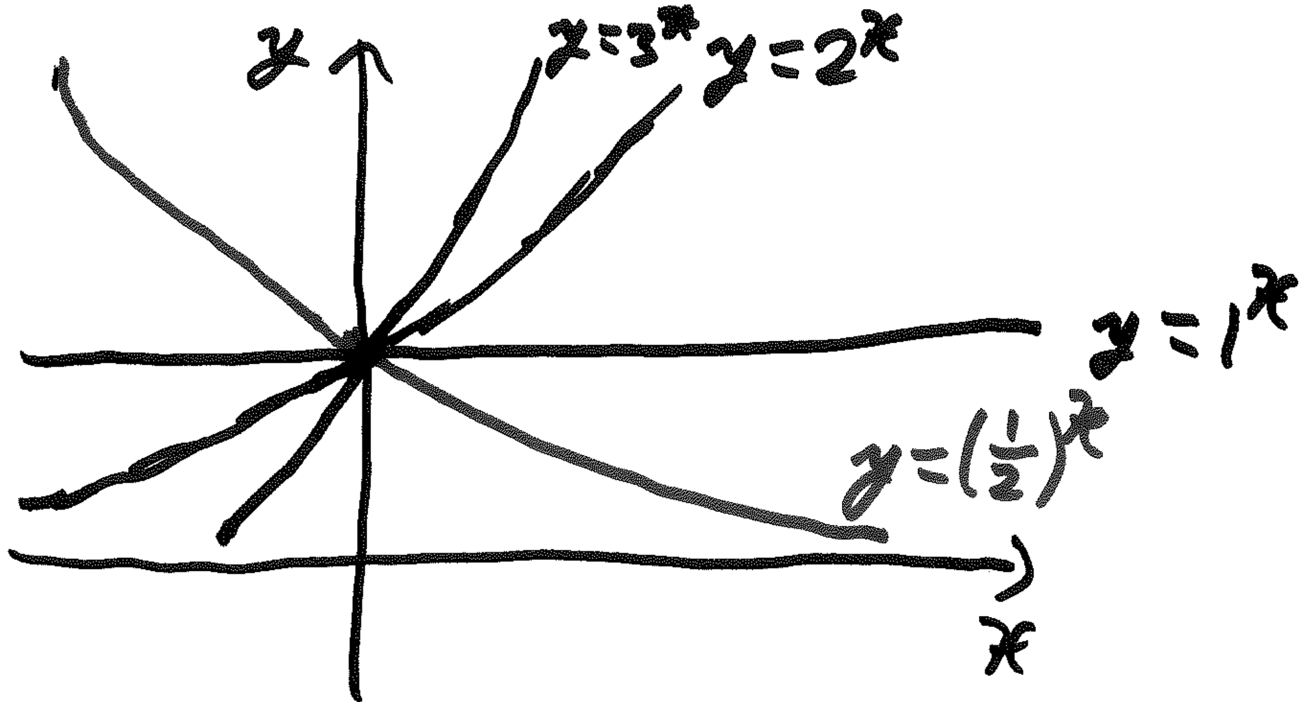
FIX $B > 0$, DEFINE $f(x) = B^x$

$f(0) = B^0 = 1$

$f(1) = B$, ... $f(n) = B^n$

$f\left(\frac{m}{n}\right) = B^{\frac{m}{n}}$

DOMAIN: $(-\infty, +\infty)$ RANGE: $[0, +\infty)$



$$f(x)=b^x \text{ IS } \begin{cases} \text{INCREASING, } b>1 \\ \text{DECREASING, } b<1 \\ \text{CONSTANT, } b=1 \end{cases}$$

<u>LAW OF EXPONENTS</u>:

$$B^{x+y} = B^x \cdot B^y$$

$$B^{x-y} = B^x / B^y$$

$$(B^x)^y = B^{xy}$$

$$(AB)^x = A^x \cdot B^x$$

CAUTION: $(A+B)^x$ NO FORMULA

SPECIAL VALUE OF B $e \approx 2.71828\ldots$

LOGARITHMS

$B>0,\ B\neq 1$

$f(x)=B^x : (-\infty,+\infty) \longrightarrow (0,+\infty)$

WHAT IS $f^{-1}(x)$?

INCREASING AND DECREASING FUNCTIONS ARE ONE-TO-ONE

$f^{-1}(x)=\log_B x : (0,+\infty) \longrightarrow (-\infty,+\infty)$

<u>OBS</u>: $\log_B 1 = 0$

$\log_B B = 1 \qquad \log_B B^r = r$

IN GENERAL:

$B^{\log_B x} = x \ , \quad x>0$

$\log_B (B^x) = x$

EX: SOLVE $3^{x^2+2x+3}=9$

$$\log_3\left(3^{x^2+2x+3}\right)=\log_3 9$$

$$x^2+2x+3=2$$

$$x^2+2x+1=0$$

$$(x+1)^2=0$$

$$x=-1$$

WHEN $b=e$, $\log_e x=\ln x$

LAWS OF LOGARITHMS

$$\log_b(xy)=\log_b x+\log_b y$$

$$\log_b\left(\frac{x}{y}\right)=\log_b x-\log_b y$$

$$\log_b(x^r)=r\log_b x$$

$$b^{\log_b(xy)}=xy=b^{\log_b x}\cdot b^{\log_b y}$$

$$=b^{\log_b x+\log_b y}$$