REVIEW

CONTINUITY (INCLUDING DISCONTINUITY, INTERMEDIATE VALUE THEOREM)

LIMITS AT INFINITY / HORIZONTAL ASYMPT.

DERIVATIVES: DEFINITION (LIMIT)

DERIVATIVE AS A FUNCTION (HIGHER DERIVS.)

DERIVATIVE RULES: SUM, DIFFERENCE, PRODUCT, QUOTIENT, CHAIN

SPECIAL FUNCTIONS: POWERS, POLYNOMIALS, RATIONAL FNS, EXPONENTIAL, TRIGONOMETRIC, INVERSE OF EXP./TRIG.

IMPLICIT DIFFERENTIATION

LOGARITHMIC DIFFERENTIATION

LIMITS AT INFINITY, HORIZ. ASYPT.

~~f.(~~ GIVEN $f(x)$, LOOK AT

$$\lim_{x\to-\infty} f(x)=L_1, \quad \lim_{x\to\infty} f(x)=L_2$$

$y=L_1$ AND $y=L_2$ HORIZ. ASYMPT.,

<u>ASSUMING</u> L_1, L_2 EXIST, FINITE

$$\lim_{x\to\infty} x^r = \begin{cases} +\infty, & r>0 \\ 0, & r<0 \\ 1, & r=0 \end{cases}$$

$$\lim_{x\to-\infty} x^n = \begin{cases} +\infty, & n \text{ EVEN POSITIVE INTEGER} \\ -\infty, & n \text{ ODD POSITIVE INTEGER} \end{cases}$$

$$\lim_{x\to-\infty} x^{-n} = 0$$

$$\lim_{x\to\infty} b^x = \begin{cases} +\infty, & b>1 \\ 0, & b<1 \\ 1, & b=1 \end{cases}$$

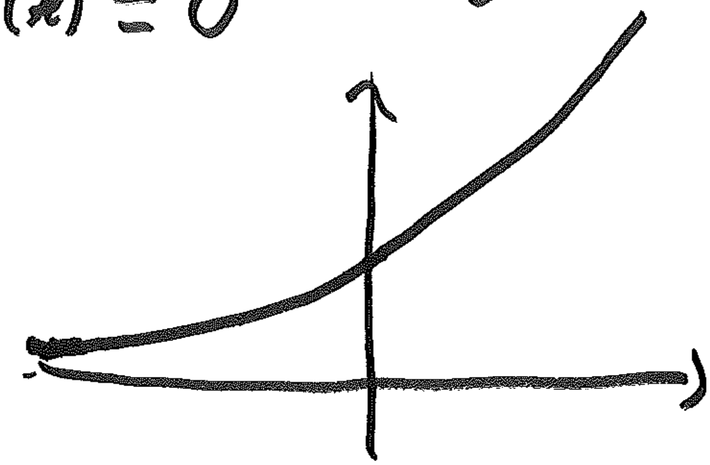
$$\lim_{x\to-\infty} b^x = \lim_{h\to+\infty} b^{-h} = \begin{cases} 0, & b>1 \\ +\infty, & b<1 \\ 1, & b=1 \end{cases}$$

<u>Ex</u>: $f(x) = e^x$ HORIZ. ASYMPT.

$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to -\infty} f(x) = 0$$

$y = 0$



<u>Ex</u>: $f(x) = \dfrac{2^x + 3^x}{1 + 5^x}$ HORIZ. ASYMPT.

$$\lim_{x \to -\infty} f(x) = \frac{\lim\limits_{x \to -\infty} (2^x + 3^x)}{\lim\limits_{x \to -\infty} (5^x + 1)} = \frac{0 + 0}{0 + 1} = \frac{0}{1} = 0$$

$$\lim_{x \to \infty} f(x) = \frac{\lim\limits_{x \to \infty} (2^x + 3^x)}{\lim\limits_{x \to \infty} (5^x + 1)} = \frac{\infty}{\infty}$$

$$f(x) = \frac{2^x + 3^x}{5^x + 1} = \frac{3^x \cdot \left(\frac{2^x}{3^x} + 1\right)}{5^x \cdot \left(1 + \frac{1}{5^x}\right)} = \left(\frac{3}{5}\right)^x \cdot \frac{\left(\frac{2}{3}\right)^x + 1}{1 + \frac{1}{5^x}}$$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \left(\frac{3}{5}\right)^x \cdot \lim_{x \to \infty} \frac{\left(\frac{2}{3}\right)^x + 1}{1 + \frac{1}{5^x}} = 0 \cdot 1 = 0$$

EX: $f(x) = \dfrac{2^x + 3^x}{1 + 3^x}$

$$\lim_{x \to -\infty} f(x) = 0$$

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{3^x}{3^x} \cdot \frac{\left(\frac{2}{3}\right)^x + 1}{1 + \left(\frac{1}{3}\right)^x} = 1 \cdot 1 = 1$$

IVT: $f : [a, b] \to \mathbb{R}$ CONTINUOUS

THEN FOR ANY N BETWEEN $f(a)$ AND $f(b)$, THERE IS SOME $a \leq c \leq b$ SO THAT $f(c) = N$

MAIN APPLICATION: APPROXIMATE SOLUTIONS TO $f(x) = 0$ / COUNT SOLUTIONS TO $f(x) = 0$

$f : [-1, 1] \to \mathbb{R}$, $f(-1) = 5$, $f(1) = 7$, $f(0) = -2$

THEN f HAS <u>AT LEAST</u> TWO ROOTS BETWEEN -1 AND 1

<u>EX</u>: $\displaystyle\lim_{x\to 0}\frac{\sin(\sin^2 x)}{3x^2}=\frac{1}{3}$

PLUG IN $x=0$ $\quad \displaystyle\frac{\sin(\sin^2 0)}{3\cdot 0^2}=\frac{0}{0}$

$$\lim_{h\to 0}\frac{\sin h}{h}=1$$

$$\frac{\sin(\sin^2 x)}{3x^2}=\frac{\sin(\sin^2 x)}{\sin^2 x}\cdot\frac{\sin^2 x}{3x^2}$$

$$\lim_{x\to 0}\frac{\sin(\sin^2 x)}{3x^2}=\underbrace{\lim_{x\to 0}\frac{\sin(\sin^2 x)}{\sin^2 x}}_{1}\cdot\lim_{x\to 0}\frac{\sin^2 x}{3x^2}$$

$$\lim_{x\to 0}\frac{\sin^2 x}{3x^2}=\frac{1}{3}\lim_{x\to 0}\frac{\sin^2 x}{x^2}=\frac{1}{3}\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2$$

$$=\frac{1}{3}\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2=\frac{1}{3}\cdot 1^2=\frac{1}{3}$$