REVIEW

DERIVATIVES

NEED TO KNOW: DEFINITION OF A DERIVATIVE AS A LIMIT

Ex: LET $f(x) = \arcsin(x^2)$

FIND $f''(x)$

$f'(x)$ CHAIN RULE

$g(x) = \arcsin x$

$h(x) = x^2$

$f'(x) = g'(h(x))\, h'(x)$

$g'(x) = \dfrac{1}{\sqrt{1-x^2}} \qquad h'(x) = 2x$

$$f'(x) = \frac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \frac{2x}{\sqrt{1-x^4}}$$

$f''(x)$ QUOTIENT RULE

$$f''(x) = \frac{(2x)'\sqrt{1-x^4} - 2x\cdot\left(\sqrt{1-x^4}\right)'}{1-x^4}$$

$$\left(\sqrt{1-x^4}\right)' \underset{\text{CHAIN RULE}}{=\!=\!=} \frac{1}{2\sqrt{1-x^4}}\cdot(1-x^4)'$$

$$= \frac{-4x^3}{2\sqrt{1-x^4}} = \frac{-2x^3}{\sqrt{1-x^4}}$$

$$f''(x) = \frac{2\sqrt{1-x^4} - 2x\cdot\frac{-2x^3}{\sqrt{1-x^4}}}{1-x^4} = \ldots$$

EX: FIND A, B SO THAT

$$f(x) = \begin{cases} A\sqrt{x+1} + Be^x & -1<x<0 \\ x^2+1 & x \geq 0 \end{cases}$$

IS DIFFERENTIABLE ON $(-1, +\infty)$

NEED TO CHECK WHAT HAPPENS WHEN $x=0$

i) WHEN IS f CONTINUOUS AT 0?

$$\lim_{x \to 0^-} f(x) = A\sqrt{0+1} + B \cdot e^0 = A+B$$

$$\lim_{x \to 0^+} f(x) = 0^2+1 = 1$$

$$f(0) = 1$$

THUS $A+B=1$, $B=1-A$

ii) f DIFFBLE AT 0

$$\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} A \cdot \frac{1}{2\sqrt{x+1}} + Be^x = \frac{A}{2} + B$$

$$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} 2x = 0$$

$$\frac{A}{2} + B = 0 \qquad \frac{A}{2} + (1-A) = 0$$

$$1 - \frac{A}{2} = 0, \quad A = 2$$

$A=2$, $B=-1$

$$B = 1-A = 1-2 = -1$$

EX: $y^5 - xy + x^2 = 3$

FIND THE TANGENT TO THE GRAPH OF y AT $(2,1)$

PLUG IN: $x=2,\ y=1$

$$1^5 - 2\cdot 1 + 2^2 = 3 \quad \checkmark$$

EQN. OF TANGENT LINE: $y - 1 = \underset{??}{m}(x-2)$

COMPUTE $\frac{dy}{dx}$ AT $x=2,\ y=1$

TAKE DERIV. WITH RESPECT TO x

$$\frac{d}{dx}\left(y^5 - xy + x^2\right) = \frac{d}{dx}3$$

$$\frac{d}{dx}y^5 = 5y^4\frac{dy}{dx}$$

$$y^5 = f(y(x)),\ f(x) = x^5$$

$$\frac{d}{dx}y^5 = f'(y(x))\cdot\frac{dy}{dx} = 5y^4\cdot\frac{dy}{dx}$$

$$\frac{d}{dx}(xy) = \left(\frac{d}{dx}x\right)y + x\frac{dy}{dx} = y + x\frac{dy}{dx}$$

$$\frac{d}{dx}(x^2) = 2x \qquad \frac{d}{dx}3 = 0$$

$$5y^4\frac{dy}{dx} - \left(y + x\frac{dy}{dx}\right) + 2x = 0$$

$$5y^4\frac{dy}{dx} - x\frac{dy}{dx} - y + 2x = 0$$

$$(5y^4 - x)\frac{dy}{dx} = y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{y-2x}{5y^4-x}}$$

PLUG IN: $x=2,\ y=1$ $\quad m = \frac{1-2\cdot 2}{5\cdot 1-2} = \frac{-3}{3} = -1$

$$y-1 = (-1)(x-2)$$

LOGARITHMS

$$\frac{d}{dx} \ln x = \frac{1}{x} \qquad x > 0$$

EX: $f(x) = \ln(\sqrt{x} \sin x) \qquad x > 0$

$f'(x) = ?$

$$\ln(\sqrt{x}\sin x) = \ln\sqrt{x} + \ln(\sin x)$$

$$= \frac{1}{2}\ln x + \ln(\sin x)$$

$$f'(x) = \left(\frac{1}{2}\ln x\right)' + [\ln(\sin x)]'$$

$$= \frac{1}{2x} + \ln'(\sin x)\cdot(\sin x)'$$

$$= \frac{1}{2x} + \frac{1}{\sin x}\cdot\cos x = \frac{1}{2x} + \cot x$$

WARNING: $\ln(x + \sqrt{x^2+1})$ CHAIN RULE

EX: $f(x)=x^2$, FIND $f'(3)$ BY USING THE DEFINITION

$f'(x)=2x \qquad f'(3)=2\cdot 3=6$

$$f'(3)=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to 0}\frac{(3+h)^2-3^2}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{3^2}+2\cdot 3\cdot h+h^2-\cancel{3^2}}{h}=\lim_{h\to 0}\frac{6h+h^2}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(6+h)}{\cancel{h}}=\lim_{h\to 0}(6+h)=6$$