Related Rates

- Given multiple variables, find ROC of one in terms of the others

Ex1: Air is pumped into a spherical balloon at a rate $50\ cm^3/s$.

How fast does the radius change when the radius is 20 cm?

Variables: V (volume), r (radius)
t (time)

Goal: $\frac{dr}{dt}$ when $r=20$ cm

Know: $V = \frac{4}{3}\pi r^3$, $\frac{dV}{dt} = 50\ cm^3/s$

Differentiate: $\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

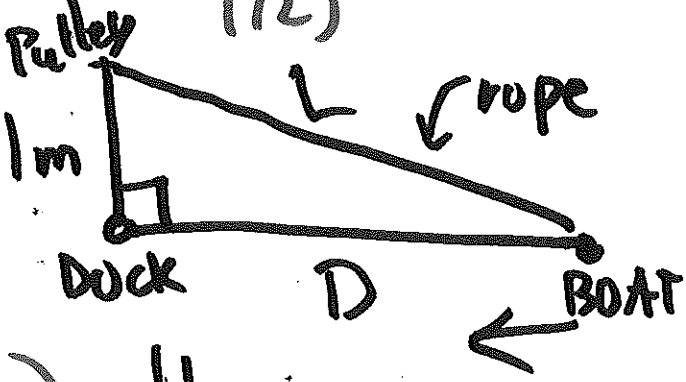
Solve: $\frac{dr}{dt} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt}$

$\frac{1}{cm^2} \cdot cm^3/s = cm/s$

$$\left.\frac{dr}{dt}\right|_{r=20} = \frac{1}{4\pi(20)^2} \cdot 50\ cm/s$$

$$= \boxed{\frac{50}{1600\pi}\ cm/s}$$

Ex2: Boat pulled to a dock by a rope. Rope is passing through a pulley 1m higher than the boat. If the rope is pulled at a rate of 1 m/s, how fast is the boat approaching when it is 8m from the dock?



D = dist. from the dock

L = length of rope

t = time

3) $\frac{dL}{dt} = -1 \text{ m/s}$ 4) $\frac{dD}{dt}\Big|_{D=8m}$

5) Pyth. Thm: $1^2 + D^2 = L^2$

6) $0 + 2D \cdot \frac{dD}{dt} = 2L \cdot \frac{dL}{dt}$

$D=8$

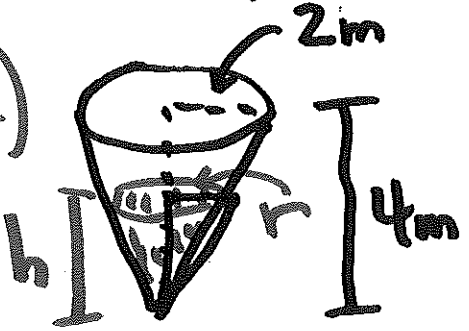
$1+64=L^2$

$L=\sqrt{65}$

7) $\frac{dD}{dt} = \frac{L}{D}\frac{dL}{dt} = \frac{\sqrt{65}}{8}(-1) = \boxed{-\frac{\sqrt{65}}{8} \text{ m/s}}$

Ex: A water tank is an inverted circular cone w/ base radius 2m & height 4m. If water is pumped in at a rate of $2\ m^3/min$, find the rate at which the water level is rising when the water is 3m deep.

1/2)



r = radius of cone of water
h = height of cone of water
V = volume of cone of water
t = time

3) $\frac{dV}{dt} = 2\ m^3/min$

4) $\left.\frac{dh}{dt}\right|_{h=3m}$

5) • similar $\triangle$s $\frac{r}{h} \sim \frac{2}{4} \Rightarrow \frac{r}{2} = \frac{h}{4}$

• $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$ $\quad r = \frac{h}{2}$

$= \frac{1}{12}\pi h^3$

6) $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \cdot \frac{dh}{dt}$ $\quad \frac{1}{12} \cdot 3 = \frac{1}{4}$

$$\frac{dh}{dt} = \frac{4}{\pi h^2} \cdot \frac{dV}{dt}$$

7) $\left.\frac{dh}{dt}\right|_{h=3} = \frac{4}{9\pi} \cdot 2 = \boxed{\frac{8}{9\pi} \text{ m/min}}$