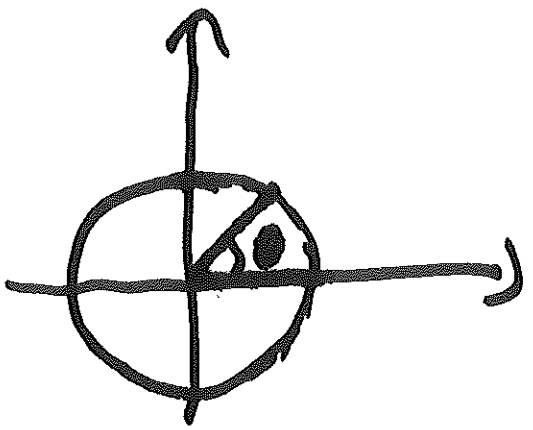
TRIGONOMETRY

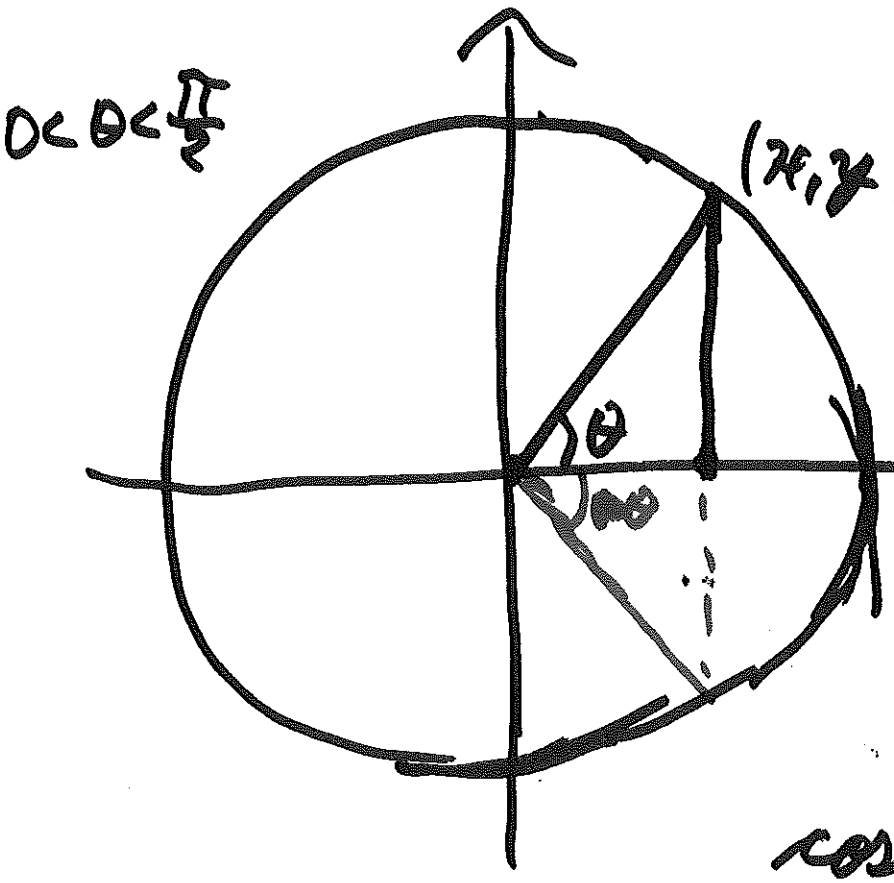


CIRCLE OF RADIUS 1

θ = ANGLE
- DEGREES
- RAD

$360^\circ \longleftrightarrow 2\pi$ RAD

$\theta^\circ \longleftrightarrow \frac{\pi\theta}{180}$ RAD



$\sin\theta = \frac{OPP}{HYP}$

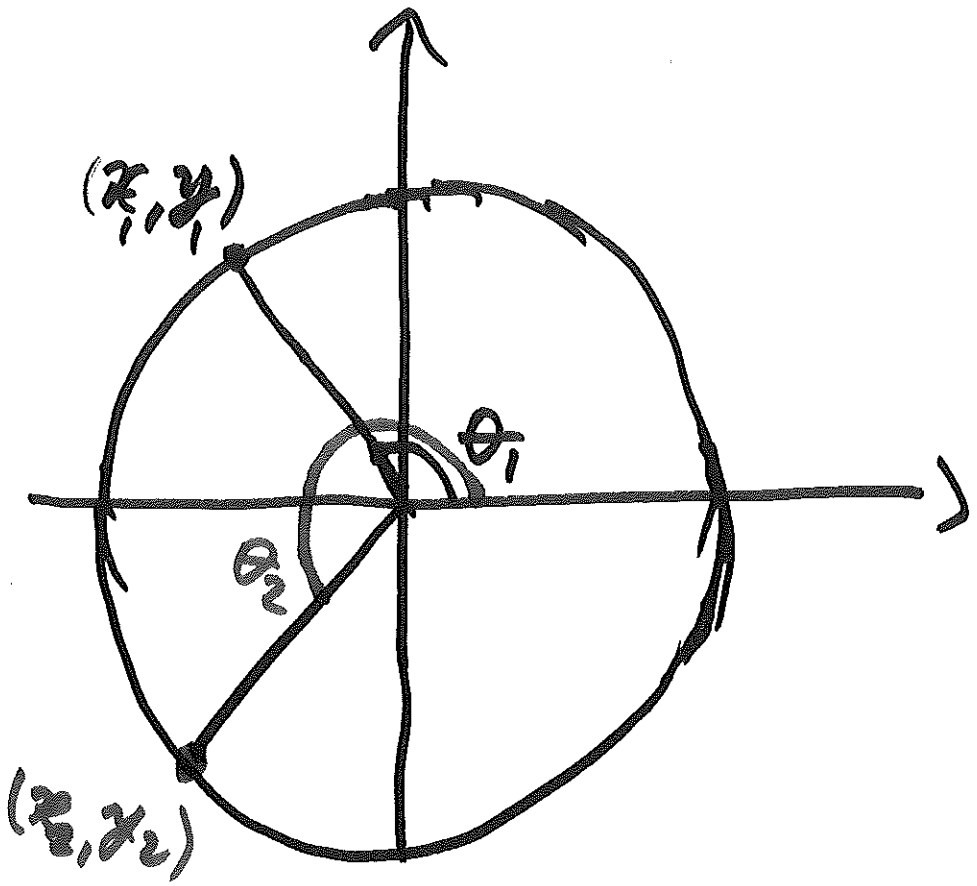
$\cos\theta = \frac{ADJ}{HYP}$

$\tan\theta = \frac{OPP}{ADJ} = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{1}{\tan\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$



$x_1 = \cos\theta_1, \; y_1 = \sin\theta_1$

$x_2 = \cos\theta_2, \; y_2 = \sin\theta_2$

CAN DEFINE $\sin, \cos$ FOR ALL $0 \le \theta < 2\pi$

NOW DEFINE $\sin, \cos$ SO THAT

$\sin(\theta + 2\pi) = \sin\theta$

$\cos(\theta + 2\pi) = \cos\theta$

PERIODIC FNS

EX: $\sin 0 = 0, \quad \cos 0 = 1$

$\sin \frac{\pi}{2} = 1, \quad \cos \frac{\pi}{2} = 0$

$\sin \pi = 0, \quad \cos \pi = -1$

$\sin(9\pi) = \sin(\pi + 4\cdot 2\pi) = \sin\pi = 0$

$\cos(9\pi) = \cos\pi = -1$

DOMAIN OF $\sin(x), \cos(x)$ IS $(-\infty, +\infty)$

DOMAIN OF $\tan(x) = \frac{\sin(x)}{\cos(x)}$ IS $\{\cos x \neq 0\}$

$$x \neq \frac{\pi}{2} + k\pi$$

RANGE OF $\sin(x), \cos(x)$ IS $[-1, 1]$

RANGE OF $\tan(x)$ IS $(-\infty, +\infty)$

TRIGONOMETRIC ~~IDENTITY~~ ID:

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$ TRUE FOR ALL θ

DIVIDE BY $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$\sin(-x) = -\sin x$ ODD FUNCTION

$\cos(-x) = \cos x$ EVEN FUNCTION

ODD: $f(-x) = -f(x)$

EVEN: $f(-x) = f(x)$

ADDITION FORMULAS

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

IN PARTICULAR: LET $x = y$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$= \cos^2 x - (1 - \cos^2 x)$$

$$= 2\cos^2 x - 1$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

GRAPHS