REVIEW

1) MATH : FINDING MAX./MIN., MVT, HOW DERIVS. AFFECT GRAPH, L'HÔPITAL

2) WORD PROBLEMS : RATES OF CHANGE, EXPONENTIAL GROWTH/DECAY, RELATED RATES, OPTIMIZATION

<u>Ex</u> : $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x + 5$

$$-4 \le x \le 4$$

i) FIND ALL LOCAL MIN/MAX

ii) FIND GLOBAL MIN/MAX

iii) FIND INFLECTION POINTS

iv) FIND INTERVALS OF MONOTONICITY AND CONCAVITY

v) FIND c SO THAT THE MVT HOLDS

i) FIND CRITICAL POINTS

f' EXISTS EVERYWHERE

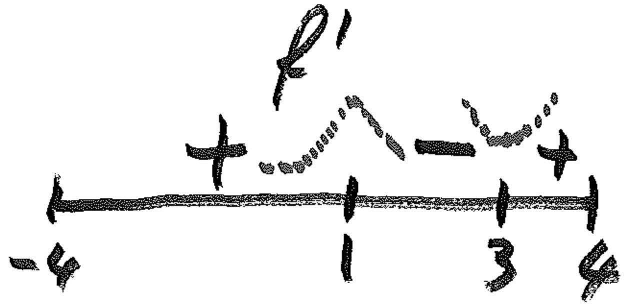
$f'(x) = x^2 - 4x + 3$

$x^2 - 4x + 3 = 0$

$x_{1,2} = \dfrac{4 \pm \sqrt{16 - 4 \cdot 3}}{2} = \dfrac{4 \pm \sqrt{4}}{2} = \dfrac{4 \pm 2}{2}$

$x_1 = 1, \; x_2 = 3$

FIRST DERIV. TEST



$-4 \leq x < 1, \; f'(x) > 0$

$1 < x < 3, \; f'(x) < 0$

$3 < x \leq 4, \; f'(x) > 0$

1 LOCAL MAX. , 3 LOCAL MIN.

SECOND DERIV. TEST

$f''(x) = 2x - 4$

$f''(1) = 2 \cdot 1 - 4 = -2 < 0$, LOCAL MAX.

$f''(3) = 2 \cdot 3 - 4 = 2 > 0$, LOCAL MIN.

ii) GLOBAL MIN/MAX

BY EVT, THEY BOTH EXIST

CANDIDATES: $x=-4$, $x=4$, $x=1$, $x=3$

$f(-4)=\frac{1}{3}(-4)^3-2\cdot(-4)^2+3\cdot(-4)+5=-\frac{181}{3}$

$f(1)=\frac{1}{3}-2+3+5=\frac{19}{3}$

$f(3)=\frac{1}{3}\cdot 3^3-2\cdot 3^2+3\cdot 3+5=5$

$f(4)=\frac{1}{3}4^3-2\cdot 4^2+3\cdot 4+5=\frac{19}{3}$

GLOBAL MIN. $x=-4$

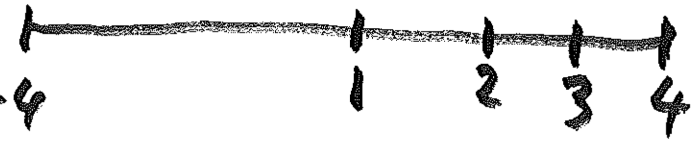
GLOBAL MAX. $x=1$, $x=4$

iii) $f''(x)=2x-4$

$2x-4=0$, $x=2$

$\forall\ -4\leq x<2$, $f''(x)<0$ CONCAVE UP

$2<x\leq 4$, $f''(x)>0$ CONCAVE DOWN

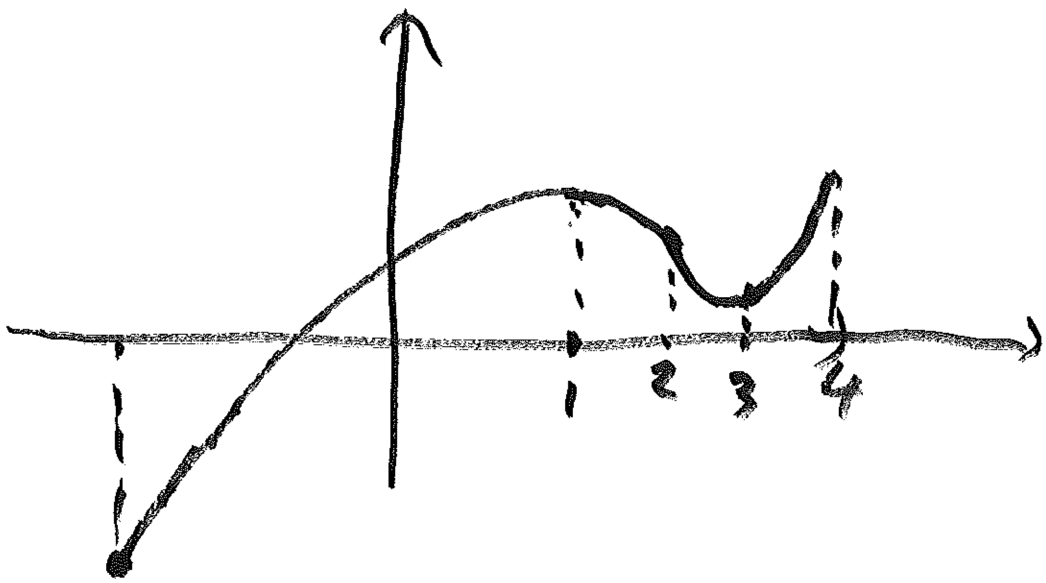
iv) 

IF $-4<x<1$, f IS INCREASING, CONCAVE UP

IF $1<x<2$, f IS DECREASING, CONCAVE UP

IF $2<x<3$, f IS DECREASING, CONCAVE DOWN

IF $3<x<4$, f IS INCREASING, CONCAVE DOWN



v) IF f ~~is~~ CTS. ON $[a, b]$,
DIFFBLE ON (a, b)

THERE IS $a<c<b$ SO THAT

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

FIND c S.T. $f'(c) = \dfrac{\frac{19}{3} - \left(-\frac{81}{3}\right)}{4-(-4)}$

$$f'(c) = \frac{200}{8 \cdot 3} = \frac{25}{3}$$

SOLVE $c^2 - 4 \cdot c + 3 = \frac{25}{3}$

$$c^2 - 4c - \frac{16}{3} = 0$$

$$c_{1,2} = \frac{4 \pm \sqrt{16 + 4 \cdot \frac{16}{3}}}{2} = \ldots$$

$$c_2 > 4$$

$$c = \frac{4 - \sqrt{16 + 4 \cdot \frac{16}{3}}}{2} = \ldots$$

L'HÔPITAL $\quad \frac{0}{0} \quad \frac{\infty}{\infty}$

$$\lim_{x \to 0} \frac{\tan x + x^2}{\sin x} \qquad \text{"}\frac{0}{0}\text{"}$$

$$\lim_{x \to 0} \frac{\tan x + x^2}{\sin x} = \lim_{x \to 0} \frac{(\tan x + x^2)'}{(\sin x)'}$$

$$= \lim_{x \to 0} \frac{\sec^2 x + 2x}{\cos x} = \frac{\sec^2 0 + 2 \cdot 0}{\cos 0} = \frac{1}{1} = 1$$

$$\lim_{x\to\infty}\frac{x^2}{e^x+x^3} \overset{L'H}{=} \lim_{x\to\infty}\frac{(x^2)'}{(e^x+x^3)'} = \lim_{x\to\infty}\frac{2x}{e^x+3x^2}$$

$$\overset{L'H}{=} \lim_{x\to\infty}\frac{2}{e^x+6x} = \frac{2}{\infty} = 0$$

$$f(x) = \frac{x^2+1}{x} = \frac{x^2}{x}+\frac{1}{x} = x+\frac{1}{x}$$

DOMAIN: $x \neq 0$ $\qquad x^{-1}$

FIND CRIT. PTS., INFLECTION POINTS

$f'(x) = 1-\frac{1}{x^2}$ CRIT. POINTS: $x^2=1$, $x=\pm 1$

$f''(x) = \frac{2}{x^3}$ INFLECTION POINTS: $\frac{2}{x^3}=0$ NONE