ANTIDERIVATIVE

<u>DEF</u>: GIVEN AN INTERVAL I, WE SAY THAT F IS AN <u>ANTIDERIVATIVE</u> OF $f(x)$ IF $F'(x)=f(x)$ FOR ALL x IN I.

ASSUME $F(x)$ AND $G(x)$ ARE BOTH ANTIDERIVS. OF $f(x)$. THEN $(F(x)-G(x))'=F'(x)-G'(x)$

$$=f(x)-f(x)=0$$

$$F(x)-G(x)=C\text{ , } C \text{ CONSTANT}$$

~~IN G~~ THE GENERAL ANTIDERIVATIVE OF A FUNCTION $f(x)$ IS OF THE FORM $F(x)+C$ FOR SOME (FIXED) ANTIDERIV. OF f

ANTIDERIVATIVE OF e^{-x^2} CANNOT BE COMPUTED FROM ELEMENTARY

FOR NOW, GUESS

$f(x)=0 \rightsquigarrow F(x)=0+C=C$

$f(x)=2 \rightsquigarrow F(x)=2x+C$

$f(x)=c$, c CONST. $\rightsquigarrow F(x)=cx+C$

$f(x)=x^n \rightsquigarrow F(x)=\frac{1}{n+1}x^{n+1}+C \quad (n\neq -1)$

$f(x)=x^{-1} \rightsquigarrow F(x)=\ln|x|+C \quad (-\infty,0)$ OR $(0,\infty)$

$f(x)=\sin x \rightsquigarrow F(x)=-\cos x+C$

$f(x)=\cos x \rightsquigarrow F(x)=\sin x+C$

$f(x)=e^x \rightsquigarrow F(x)=e^x+C$

$f(x)=b^x,\ b>0 \rightsquigarrow F(x)=\frac{b^x}{\ln b}+C,\ b\neq 1$

$f(x)=\sec^2 x \rightsquigarrow F(x)=\tan x+C$

$f(x)=\frac{1}{\sqrt{1-x^2}} \rightsquigarrow F(x)=\arcsin x+C$

$f(x)=\frac{1}{1+x^2} \rightsquigarrow F(x)=\arctan x+C$

ANTIDERIVATIVE RULES

ASSUME $F' = f$, $G' = g$, c CONSTANT

$$cf \rightsquigarrow cF + C$$

$$f + g \rightsquigarrow F + G + C$$

<u>EX</u>: $0 < x < +\infty$, $f(x) = \frac{x^2+1}{x} + 5 \cdot 3^x + \frac{2}{1+x^2}$

FIND ALL ANTIDERIVATIVES OF f

FIND $F_1' = \frac{x^2+1}{x}$, $F_2' = 5 \cdot 3^x$, $F_3' = \frac{2}{1+x^2}$

$$F_3(x) = 2 \arctan x$$

$$F_2(x) = 5 \cdot \frac{3^x}{\ln 3}$$

$$\frac{x^2+1}{x} = \frac{x^2}{x} + \frac{1}{x} = x + \frac{1}{x}$$

$$F_1(x) = \frac{x^2}{2} + \ln x$$

$$F(x) = \frac{x^2}{2} + \ln x + \frac{5}{\ln 3} \cdot 3^x + 2 \arctan x + C$$

TO DETERMINE C, WE NEED TO KNOW $F(x_0)$ FOR SOME x_0

EX: $f(x) = 2x+3$

$$F(x) = x^2+3x+C$$

IF $F(1)=5$, PLUG IN $x=1$

$$1^2+3\cdot 1+C=5, \quad C=1$$

<u>APPLICATION</u>: GIVEN $a(t)$ = ACCELERATION AT TIME t, COMPUTE VELOCITY, POSITION

EX: ASSUME A PARTICLE STARTS AT THE ORIGIN ~~WITH AN~~ AT REST, AND ITS ACCEL. IS $a(t) = t^2+1$. WHAT IS THE POSITION AT $t=2$?

$v(t)$ = VELOCITY, $s(t)$ = POSITION

$$s'(t) = v(t), \quad v'(t) = a(t)$$

$$v'(t) = t^2+1 \qquad v(t) = \tfrac{1}{3}t^3+t+C_1$$

$$v(0)=0 \qquad 0 = \tfrac{1}{3}0^3+0+C_1, \quad C_1=0$$

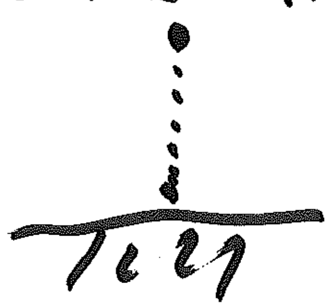
$s'(t) = \frac{1}{3}t^3 + t$

$s(t) = \frac{1}{12}t^4 + \frac{t^2}{2} + C_2$

$s(0) = 0 \qquad C_2 = 0$

$s(t) = \frac{1}{12}t^4 + \frac{t^2}{2} \qquad s(2) = \frac{16}{12} + 2$

<u>EX</u>: THROW A STONE WITH INITIAL VELOCITY 20 m/s. ASSUME $g = 10\ m/s^2$. WHAT IS THE MAXIMUM HEIGHT THE STONE REACHES.



$s(t)$ = HEIGHT AT TIME t

KNOW: $s''(t) = -10$

$s(0) = 0, \quad s'(0) = 20$

$v(t) = s'(t)$

$v'(t) = -10, \quad v(t) = -10t + C_1$

$20 = v(0) = -10 \cdot 0 + C_1, \quad C_1 = 20$

$s'(t) = 20 - 10t$

$s(t) = 20t - 5t^2 + C_2 \qquad s(0) = 0, \quad C_2 = 0$

$s(t) = 20t - 5t^2$