INDEFINITE INTEGRAL AND NET CHANGE THEOREM

RECALL: FTC II

f CONTINUOUS ON $[a,b]$, THEN

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

F IS ANY ANTIDERIVATIVE

$\int_a^b f(x)\,dx$ IS A DEFINITE INTEGRAL

(A NUMBER)

$F(x) = \int f(x)\,dx$ IS THE INDEFINITE INTEGRAL

(A FUNCTION)

FAMILY OF ALL ANTIDERIVATIVES OF f

EX: $\int x^3\,dx = \frac{x^4}{4} + C$

$\int \cos x\,dx = \sin x + C$

VS.

$\int_0^{2\pi} \cos x\,dx = \sin(2\pi) - \sin 0 = 0 - 0 = 0$

COMPUTING INDEFINITE/DEFINITE INTEGRALS

USE: $\int f(x)+g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

$\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

SOMETIMES: SIMPLIFY

EX: $\int \frac{x^2+1}{\sqrt{x}}\,dx = \int \frac{x^2}{\sqrt{x}} + \frac{1}{\sqrt{x}}\,dx$

$= \int x^{\frac{3}{2}} + x^{-\frac{1}{2}}\,dx = \int x^{\frac{3}{2}}\,dx + \int x^{-\frac{1}{2}}\,dx$

$= \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{1}{2}} + C \qquad F(x) = \frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{1}{2}}$

$\int_1^2 \frac{x^2+1}{\sqrt{x}}\,dx = F(2) - F(1) = \frac{2}{5}2^{\frac{5}{2}} + 2\cdot 2^{\frac{1}{2}} - \frac{2}{5} - 2$

$\int_0^2 \frac{x^2+1}{\sqrt{x}}\,dx \stackrel{?}{=} F(2) - F(0)$ DOES NOT APPLY

$f(x) = \frac{x^2+1}{\sqrt{x}}$ NOT CONTINUOUS AT 0

<u>RECALL</u>: DERIVATIVE IS THE RATE OF CHANGE OF A FUNCTION

THE NET CHANGE OF A FUNCTION F BETWEEN a AND b IS $F(b)-F(a)$

<u>NET CHANGE THEOREM</u>:

$$\int_a^b \text{RATE OF CHANGE} = \text{NET CHANGE}$$

$$\int_a^b F'(x)\,dx = F(b)-F(a) \quad \text{FTC } \underline{\text{II}}$$

<u>EX</u>: A POPULATION OF SHARKS INCREASES AT A RATE OF t^2+2t PER YEAR. IF WE START WITH 100 SHARKS, HOW MANY SHARKS WILL WE HAVE AFTER TEN YEARS?

WE KNOW: $F(0)=100$; NEED TO KNOW $F(10)$

$$F'(t)=t^2+2t$$

$$F(10)-F(0)=\int_0^{10} t^2+2t\,dt \qquad \int t^2+2t\,dt = \frac{t^3}{3}+t^2+C$$

$$F(10)=100+\left(\frac{10^3}{3}+10^2\right)-\left(\frac{0^3}{3}+0^2\right)=200+\frac{1000}{3}$$

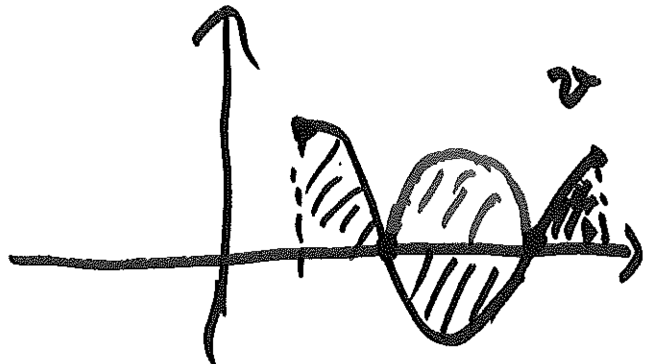
EX: ASSUME WE KNOW THE VELOCITY $v(t)$ AT EACH TIME

i) FIND THE POSITION $s(t)$

ii) FIND THE TOTAL DISTANCE TRAVELLED

FOR i): $s(b) - s(a) = \int_a^b v(t)\,dt$

FOR ii): TOTAL DIST. $= \int_a^b |v(t)|\,dt$



EX: $v(t) = t^2 - 4t + 3$, $0 \leq t \leq 4$

$s(0) = 0$

i) WHAT IS $s(4)$?

$s(4) - s(0) = \int_0^4 t^2 - 4t + 3\,dt$

$\int t^2 - 4t + 3\,dt = \frac{t^3}{3} - 2t^2 + 3t + C$

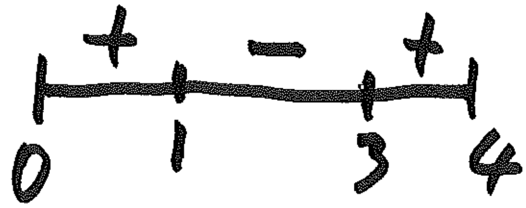
$C = 0$

$$\int_0^4 t^2-4t+3\,dt = \left(\frac{4^3}{3}-2\cdot4^2+3\cdot4\right)-\left(\frac{0^3}{3}-2\cdot0^2+3\cdot0\right)$$
$$= \frac{64}{3}-32+12 = \ldots$$

ii) WHAT IS THE TOTAL DISTANCE TRAVELLED?

SOLVE $v(t)=0$

$$t_1, t_2 = \frac{4 \pm \sqrt{4^2-4\cdot3}}{2} \qquad t_1=1,\ t_2=3$$



$$\int_0^4 |v(t)|\,dt = \int_0^1 v(t)\,dt + \int_1^3 -v(t)\,dt$$
$$+ \int_3^4 v(t)\,dt$$

$$\int_0^1 t^2-4t+3\,dt = \frac{1}{3}-2+3 = \frac{4}{3}$$

$$-\int_1^3 t^2-4t+3\,dt = -\left[\left(\frac{3^3}{3}-2\cdot3^2+3\cdot3\right)-\left(\frac{1}{3}-2+3\right)\right]$$
$$= -\left[\underbrace{(9-18+9)}_{0}-\frac{4}{3}\right] = -\left(-\frac{4}{3}\right)$$
$$= \frac{4}{3}$$

$$\int_3^4 t^2-4t+3\,dt = \left[\frac{4^3}{3}-2\cdot4^2+3\cdot4\right]-\left[\frac{3^3}{3}-2\cdot3^2+3\cdot3\right]$$
$$= \frac{64}{3}+12-32$$

TOTAL DIST. $= \frac{4}{3} + \frac{4}{3} + \frac{64}{3} - 20$