METHOD OF SUBSTITUTION

INDEFINITE INTEGRALS

CHAIN RULE $\left[F(g(x))\right]' = F'(g(x))\,g'(x)$

$$\int F'(g(x))\,g'(x)\,dx = \int \left(F(g(x))\right)' dx = F(g(x)) + C$$

<u>EX</u>: $\int \cos(x^2)\cdot 2x\,dx = \sin x^2 + C$

$F(x) = \sin x$, $g(x) = x^2$

A DIFFERENT APPROACH

$F' = f$

$$\int f(g(x))\,g'(x)\,dx = F(g(x)) + C$$

LET $u = g(x)$ THEN $\frac{du}{dx} = g'(x)$

$du = g'(x)\,dx$

$$\boxed{\int f(g(x))\,g'(x)\,dx = \int f(u)\,du}$$

EX: $\int \cos(x^2)\cdot 2x\,dx = \int \cos u\,du = \sin u + C$

LET $u = x^2$, $du = 2x\,dx$ $\qquad = \sin x^2 + C$

EX: $\int \sin(5x)\,dx = \int (\sin u)\cdot\frac{1}{5}\,du = \frac{1}{5}\int \sin u\,du$

LET $u = 5x$, $du = (5x)'\,dx = 5\,dx$

$dx = \frac{1}{5}\,du$

$= \frac{1}{5}(-\cos u) + C = -\frac{1}{5}\cos(5x) + C$

EX: $\int (3x-2)^{50}\,dx =$

LET $u = 3x-2$, $du = (3x-2)'\,dx = 3\,dx$

$dx = \frac{1}{3}\,du$

$= \int u^{50}\cdot\frac{1}{3}\,du = \frac{1}{3}\int u^{50}\,du = \frac{1}{3}\cdot\frac{u^{51}}{51} + C$

$= \frac{(3x-2)^{51}}{153} + C$

TRY TO MAKE COMPLICATED EXPRESSIONS SIMPLER

EX: $\int (\sin x)(\cos x)^{10}\,dx =$

$u = \sin x$, $du = (\cos x)\,dx$

DOES NOT BECOME SIMPLER!

$u = \cos x$, $du = -(\sin x)\,dx$

$-du = (\sin x)\,dx$

$$= \int u^{10}\cdot(-du) = -\frac{u^{11}}{11} + C = -\frac{(\cos x)^{11}}{11} + C$$

EX: $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$

$u = \cos x$, $du = -(\sin x)\,dx$

$$\int \tan x\,dx = \int \frac{-du}{u} = -\int \frac{1}{u}\,du = -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

GOOD IDEAS FOR SUBSTITUTION: QUANTITIES INSIDE ROOTS, POWERS, TRIG. FNS., DENOMINATORS ETC.

DEFINITE INTEGRALS

$$\int_a^b f(g(x))\,g'(x)\,dx = F(g(b)) - F(g(a))$$

BY FTC II $\qquad F' = f$

TWO WAYS:

i) FIND AN ANTIDERIVATIVE, EVALUATE

ii) $u = g(x)$

$$\boxed{\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du}$$

NEED TO CHANGE BOUNDS OF INTEGRATION

EX: $\int_0^1 x(6x^2+1)^5\,dx = F(1) - F(0) = \frac{7^6}{72} - \frac{1}{72}$

i) $\int x(6x^2+1)^5\,dx = \int u^5 \cdot \frac{1}{12}\,du =$

$u = 6x^2+1$, $du = (6x^2+1)'\,dx = 12x\,dx$

$x\,dx = \frac{1}{12}\,du$

$= \frac{1}{12}\frac{u^6}{6} = \frac{(6x^2+1)^6}{72} = F(x)$

ii) $u = 6x^2+1$ $\quad g(0) = 6\cdot 0^2+1 = 1$

$g(1) = 6\cdot 1^2+1 = 7$

$$\int_0^1 x(6x^2+1)^5\,dx = \int_1^7 u^5\cdot \frac{1}{12}\,du$$

$$= \frac{1}{12}\cdot\left(\frac{7^6}{6} - \frac{1^6}{6}\right) = \frac{7^6}{72} - \frac{1}{72}$$

EVEN/ODD FUNCTIONS

f EVEN IF $f(x) = f(-x)$

f ODD IF $f(-x) = -f(x)$

EX: EVEN POWERS, $\cos x$, $\sqrt{1+x^2}$ ETC.

ODD POWERS, $\sin x$, ETC.

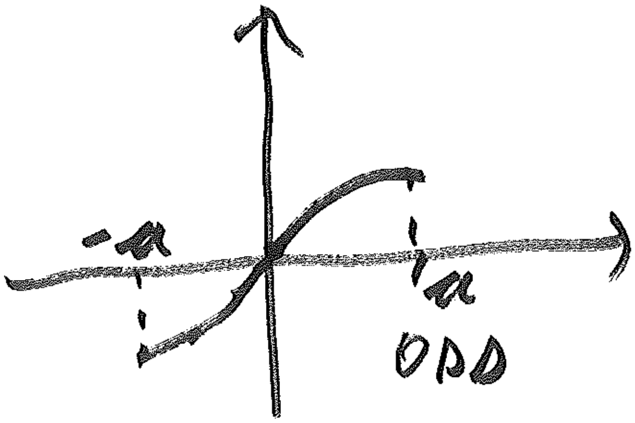
EVEN + EVEN = EVEN

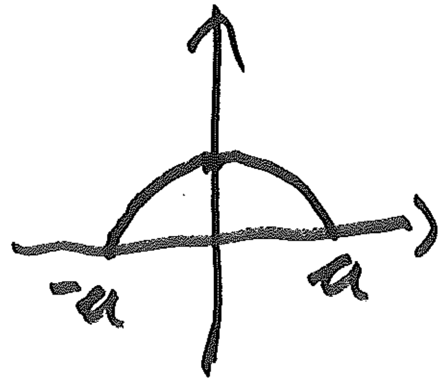
EVEN · EVEN = EVEN

ODD · EVEN = ODD

IF f IS ODD, $\int_{-a}^{a} f(x)\,dx = 0$

IF f IS EVEN, $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$





$$\int_{-a}^{a} f(x)\,dx = \int_{a}^{-a} f(-u)\cdot(-du)$$

$$= \int_{-a}^{a} f(-u)\,du$$

$u = -x, \quad x = -u$

$dx = -du$

f ODD $\quad f(-u) = -f(u)$

$$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{a} -f(u)\,du = -\int_{-a}^{a} f(u)\,du$$

$$\int_{-a}^{a} f(x)\,dx = 0$$