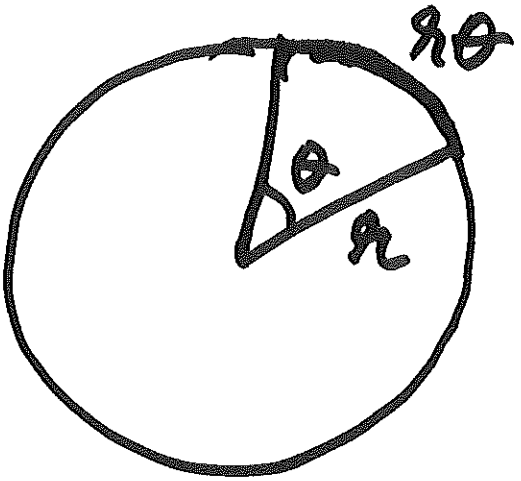
TRIGONOMETRIC FUNCTIONS



$$\sin\frac{\pi}{3} = \frac{1}{2}$$

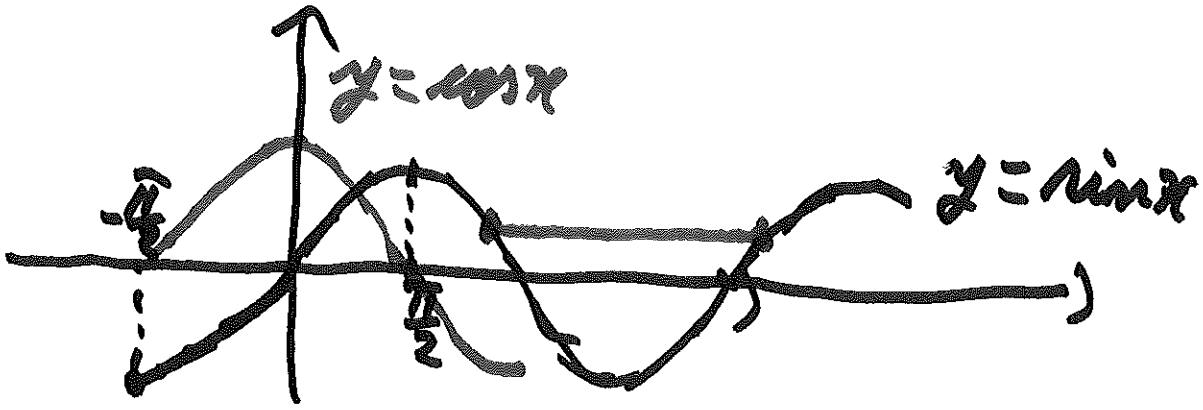
$$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

INVERSE TRIG FUNCTIONS

$\sin, \cos : (-\infty, +\infty) \longrightarrow [-1, 1]$



LOOK AT $\sin : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, 1]$

THIS IS ONE-TO-ONE

BY DEFINITION, $\sin^{-1} : [-1, 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\sin^{-1} x = y$ EXACTLY WHEN $\sin y = x$

LOOK AT $\cos : [0, \pi] \to [-1, 1]$

THIS IS ONE-TO-ONE

BY DEFINITION, $\cos^{-1} : [-1, 1] \to [0, \pi]$

$\cos^{-1} x = y$ EXACTLY WHEN $x = \cos y$

NOTE: $\sin(\sin^{-1} x) = x, \quad -1 \le x \le 1$

$\sin^{-1}(\sin x) = x, \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\sin^{-1}(\sin \pi) = \sin^{-1}(0) = 0$

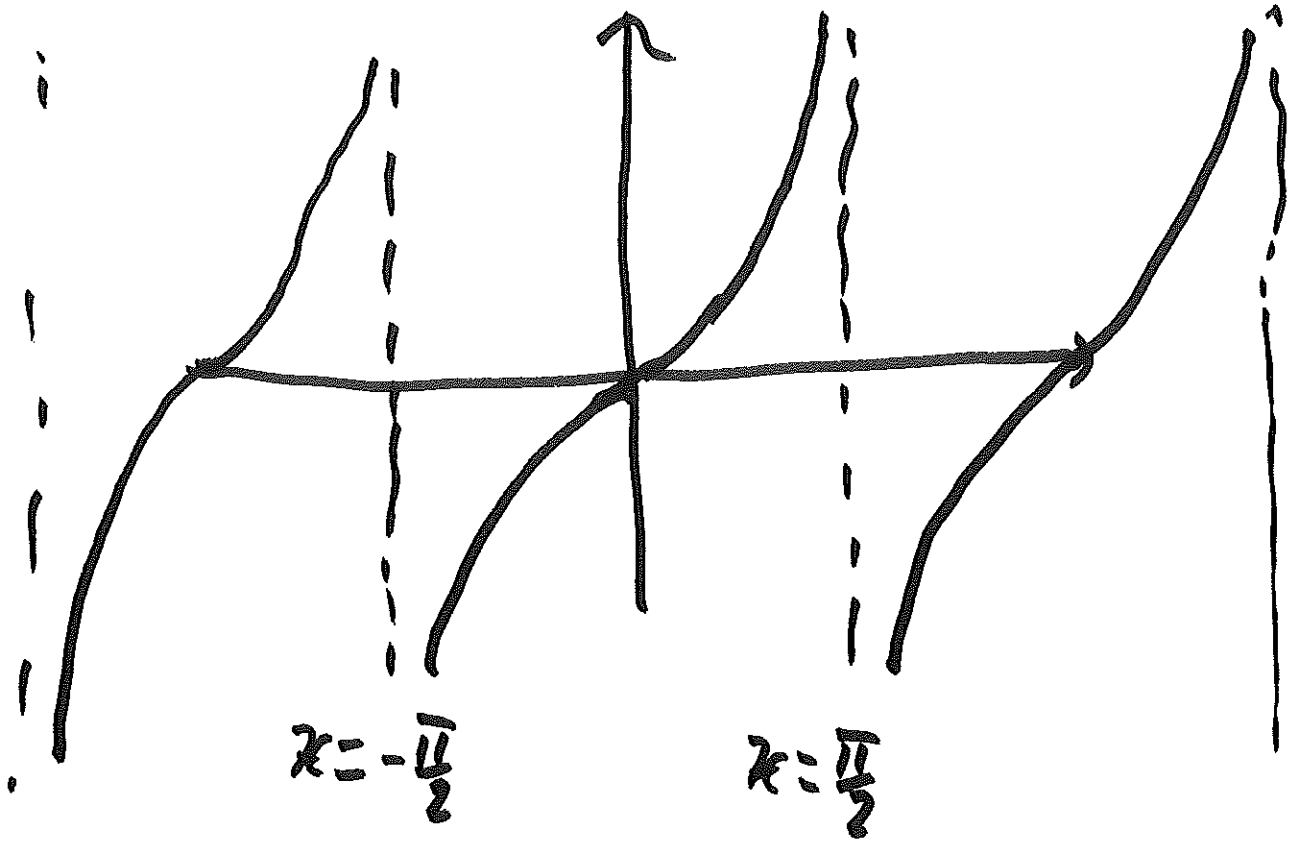
IF NOT TRUE THAT $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$,

MUST SUBTRACT THE CORRECT MULTIPLE OF π

$\cos(\cos^{-1} x) = x, \quad -1 \le x \le 1$

$\cos^{-1}(\cos x) = x, \quad 0 \le x \le \pi$

$\tan: x \neq \frac{\pi}{2} + k\pi$



$\tan: (-\frac{\pi}{2}, \frac{\pi}{2}) \to (-\infty, +\infty)$

$\tan^{-1}: (-\infty, +\infty) \to (-\frac{\pi}{2}, \frac{\pi}{2})$

$\tan^{-1} x = y$ IF $\tan y = x$

SIMPLIFYING EXPRESSIONS

$\cos(\sin^{-1} x) = \sqrt{1-x^2}$

DOMAIN : $[-1, 1]$

$\sin^{-1} x = y, \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\sin y = x$

$\cos(\sin^{-1} x) = \cos y$

RECALL:

$\sin^2 y + \cos^2 y = 1$

$x^2 + \cos^2 y = 1$

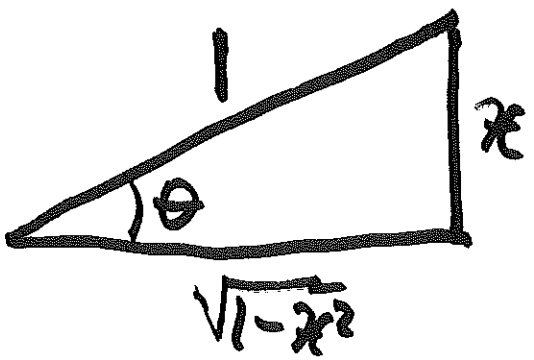
$\cos^2 y = 1 - x^2$

$\cos y = \pm\sqrt{1-x^2}$

SINCE $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, MUST HAVE

$\cos y = \sqrt{1-x^2}$

GEOMETRICALLY



$\theta = \sin^{-1} x$

$\sin\theta = x$

$\cos\theta = \sqrt{1-x^2}$

BY PYTHAGORA'S

REVIEW (NO CALCULUS)

FUNCTIONS DOMAIN + RANGE

OPERATIONS

COMPOSITION

INVERSE

EXAMPLES: POLYNOMIALS, RATIONAL, EXPONENTIALS, TRIGONOMETRIC, INVERSES