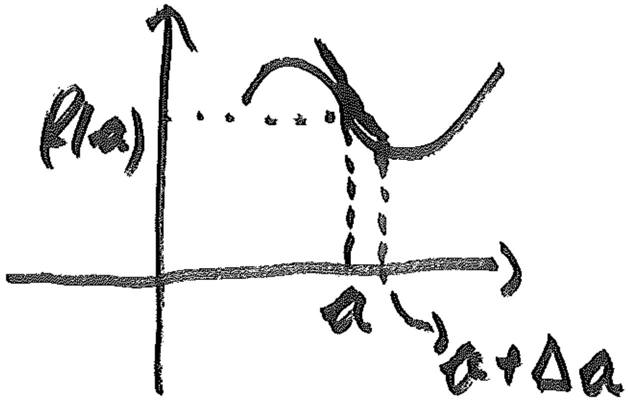
LINEAR APPROXIMATION



WE KNOW $f(a)$
WANT $f(a+\Delta a)$
APPROXIMATELY

RECALL: $f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$

WHEN CLOSE TO a, $\frac{f(x)-f(a)}{x-a} \approx f'(a)$

$f(x) - f(a) \approx f'(a)(x-a)$

$\boxed{f(x) \approx f(a) + f'(a)(x-a)}$ WHEN x IS CLOSE TO a

LINEAR APPROXIMATION

$L(x) = f(a) + f'(a)(x-a)$ LINEARIZATION OF f NEAR a

GIVEN f FUNCTION, a NUMBER, COMPUTE $f(a)$, $f'(a)$, THEN $L(x) = f(a) + f'(a)(x-a)$

<u>EX 1</u> : $f(x) = 2x+1$

LINEARIZATION OF f AT a

TANGENT LINE TO THE GRAPH AT $(a, f(a))$

$$L(x) = f(a) + f'(a)(x-a)$$

$$= 2a+1+2(x-a) = \cancel{2a}+1+2x-\cancel{2a}$$

$$= 2x+1$$

<u>EX 2</u> : $f(x) = \sqrt{x+2}$

LINEARIZATION OF f AT 0, AT 2

$$f'(x) = \frac{1}{2\sqrt{x+2}}$$

WHEN $a=0$: $L(x) = \sqrt{2} + \frac{1}{2\sqrt{2}}x$

$f(0) = \sqrt{2}$

$f'(0) = \frac{1}{2\sqrt{2}}$

WHEN $a=2$: $L(x) = 2 + \frac{1}{4}(x-2) = \frac{1}{4}x + \frac{3}{2}$

$f(2) = 2$

$f'(2) = \frac{1}{4}$

EX 3: $f(x)=\dfrac{1}{1-x}$ LINEARIZATION NEAR $a=0$?

$$f'(x)=\left((1-x)^{-1}\right)'=(-1)(1-x)^{-2}\cdot(1-x)'=\frac{1}{(1-x)^2}$$

$$f(0)=\frac{1}{1-0}=1$$

$$f'(0)=\frac{1}{(1-0)^2}=1$$

$$L(x)=f(0)+f'(0)x=1+x$$

APPLICATIONS

$f(x)=(1+x)^K$, LINEARIZATION WHEN $a=0$

$$f'(x)=K(1+x)^{K-1}$$

$$f(0)=1,\ f'(0)=K$$

$$(1+x)^K\approx 1+Kx$$

$$(1.01)^5\approx 1+5\cdot .01=1.05$$

$x=.01$ $\qquad 1.01^5=1.0510100501$

CAREFUL:

$(1.01)^{100}$

1) THIS IS $\approx 1^{100} = 1$

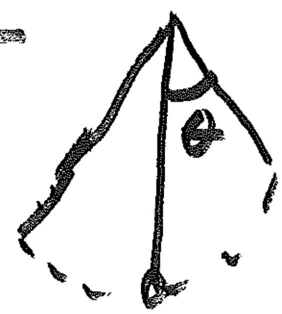
2) LINEAR APPROX.

$\approx 1 + 100 \cdot .01 = 2$

3) ACTUAL ANSWER 2.70

PHYSICS EXAMPLE

$\sin\theta \approx \theta$



LINEAR APPROX. OF $\sin x$ NEAR $a=0$

$\sin 0 = 0$

$(\sin x)' = \cos x$ $\qquad \cos 0 = 1$

x CLOSE TO 0: $\sin x \approx \sin 0 + (\cos 0)\cdot x$
$= x$

LINEAR APPROX. OF $\cos x$ NEAR $a=0$

$\cos 0 = 1$ $\qquad (\cos x)' = -\sin x$

$\cos x \approx \cos 0 + (-\sin 0)\cdot x = 1$

COMPUTING SQUARE ROOTS

$\sqrt{24} \approx \sqrt{25} = 5$

LOOK AT $f(x) = \sqrt{x}$

COMPUTE $L(x)$ AT $a = 25$

$f'(x) = \frac{1}{2\sqrt{x}}$

$L(x) = f(a) + f'(a)(x-a)$

$= 5 + \frac{1}{10}(x-25)$

PICK $x = 24$

$f(24) \approx L(24) = 5 + \frac{1}{10}(24-25) = 5 - \frac{1}{10} = 4.9$

<u>EX</u>: $f(x) = x^3 - x^2 + 5x - 7$

$f(2.01)$ 2.01 IS CLOSE TO 2

$f(2.01) \approx f(2) = 8 - 4 + 10 - 7 = 7$

BETTER: $f(2.01) \approx f(2) + f'(2) \cdot .01$

$f'(x) = 3x^2 - 2x + 5$ $f'(2) = 12 - 4 + 5 = 13$

$f(2.01) \approx 7 + 13 \cdot .01 = 7.13$

METHOD OF SUBSTITUTION

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^2 x \cdot \cos x \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1}{\sin^2 x} \cos x \, dx$$

$$\int F(\sin x) \cdot \cos x \, dx = \int F(u) \, du$$

$u = \sin x \quad du = (\sin x)' \, dx = \cos x \, dx$

$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \sin \frac{\pi}{2} = 1$

$$= \int_{\frac{\sqrt{2}}{2}}^{1} \frac{1}{u^2} \, du = -u^{-1} \Big|_{\frac{\sqrt{2}}{2}}^{1} = (-1^{-1}) - \left(-\left(\tfrac{\sqrt{2}}{2}\right)^{-1}\right)$$

$$= -1 + \sqrt{2} = \sqrt{2} - 1$$

$$f(u) \Big|_a^b = f(b) - f(a)$$