REVIEW 1:

NEW MATERIAL

ANTIDERIVATIVES

DEFINITE INTEGRALS (AREA / DISTANCE)

FTC I + II, APPLICATIONS

METHOD OF SUBSTITUTIONS

LINEAR APPROXIMATION

HIGHER ORDER APPROXIMATION

NO RIEMANN SUMS

EX: LET $f(x) = x^2 e^{x^3}$

1) FIND ITS ANTIDERIVATIVE

2) COMPUTE $\int_0^2 f(x)\,dx$

3) FIND THE LINEAR APPROXIMATION OF $f(x)$ NEAR $a=1$

4) FIND THE TAYLOR POLYNOMIAL $T_2(x)$ NEAR $a=1$

1) $F(x) = \int x^2 e^{x^3} dx$

TRY $u = x^3$ $\quad du = (x^3)' dx = 3x^2 dx$

$$x^2 dx = \frac{du}{3}$$

$$\int x^2 e^{x^3} dx = \int e^u \cdot \tfrac{1}{3} du$$

$$= \tfrac{1}{3} \int e^u du = \tfrac{1}{3} e^u + C = \tfrac{1}{3} e^{x^3} + C$$

2) $\int_0^2 f(x)\,dx \overset{FTC\,2}{=\!=} F(2) - F(0) = \tfrac{1}{3}(e^8 - 1)$

F IS $\underbrace{\text{ANY}}$ ANTIDERIVATIVE

$F(x) = \tfrac{1}{3} e^{x^3}$

$F(2) = \tfrac{1}{3} e^8$, $F(0) = \tfrac{1}{3}$

$$\int_0^2 x^2 e^{x^3} dx = \int_0^8 e^u \cdot \tfrac{1}{3} du = \tfrac{1}{3} e^u \Big|_0^8 = \tfrac{1}{3} e^8 - \tfrac{1}{3}$$

$u = x^3$

3), 4) n^{TH} ORDER TAYLOR POLYNOMIAL NEAR a

$$T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

3) $T_1(x) = f(a) + f'(a)(x-a)$

$a=1 \quad f(x) = x^2 e^{x^3} \quad f(1) = e$

$f'(x) = (x^2 e^{x^3})' = (x^2)' e^{x^3} + x^2 (e^{x^3})'$

$= 2x e^{x^3} + x^2 \cdot e^{x^3} \cdot 3x^2 = (2x + 3x^4) e^{x^3}$

$f'(1) = (2+3) e^1 = 5e$

$T_1(x) = e + 5e(x-1)$

4) $T_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$

$f''(x) = \left[(2x+3x^4)e^{x^3}\right]' = (2x+3x^4)' e^{x^3}$

$+ (2x+3x^4)(e^{x^3})' = (2+12x^3)e^{x^3} + (2x+3x^4) \cdot$

$\cdot 3x^2 e^{x^3} = (2+12x^3+6x^3+9x^6) e^{x^3}$

$= (2+18x^3+9x^6) e^{x^3}$

$f''(1) = (2+18+9) e = 29e$

$T_2(x) = e + 5e(x-1) + \frac{29e}{2}(x-1)^2$

Ex: $F(x)=\int_{\sin x}^{x^2+1}\ln(1+t^4)\,dt$

$F'(x)=?$

$F(x)=\int_{\sin x}^{0}\ln(1+t^4)\,dt+\int_{0}^{x^2+1}\ln(1+t^4)\,dt$

$=-\int_{0}^{\sin x}\ln(1+t^4)\,dt+\int_{0}^{x^2+1}\ln(1+t^4)\,dt$

$F'(x)=-\ln(1+\sin^4 x)\cdot\cos x$

$+\ln\left(1+(x^2+1)^4\right)\cdot 2x$

Ex: $\int\frac{\cos(\ln x)}{x}\,dx$ $\quad x>0$

$(\ln x)'=\frac{1}{x}$

$u=\ln x$, $du=(\ln x)'dx=\frac{1}{x}dx$

$\int\frac{\cos(\ln x)}{x}\,dx=\int\cos u\,du=\sin u+C$

$=\sin(\ln x)+C$