REVIEW 2

OLD MATERIAL:

FUNCTIONS

LIMITS: COMPUTE, PROPERTIES, LEFT/RIGHT, L'HOPITAL

CONTINUITY: <u>DEFINITION</u>, CHECK f CONTINUOUS

DERIVATIVES: DEFINITION, COMPUTE, RULES (PRODUCT, CHAIN ETC.), IMPLICIT DIFF., TANGENT LINE

APPLICATIONS: EXTREMA OF FUNCTIONS, OPTIMIZATION PROBLEMS

CONTINUITY

<u>DEF</u>: f IS CONTINUOUS AT a IF

$\lim_{x \to a} f(x) = f(a)$ PROVIDED THE LIMIT EXISTS

f IS CONTINUOUS ON AN INTERVAL (a, b) IF f CONTINUOUS AT EVERY POINT $a < c < b$

<u>EX</u>: FIND CONSTANTS a, b SO THAT THE FUNCTION

$$f(x) = \begin{cases} ax+3, & 0 < x < 1 \\ b, & x = 1 \\ \frac{x-1}{x^2-1}, & x > 1 \end{cases}$$

IS CONTINUOUS ON $(0, +\infty)$

IF $0 < c < 1$, f IS CONTINUOUS AT c

IF $1 < c$, f IS CONTINUOUS AT c

WHAT HAPPENS IF $c = 1$?

WANT: $\lim\limits_{x \to 1} f(x) = f(1)$

$f(1) = b$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = b$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (ax + 3) = a \cdot 1 + 3 = a + 3$$

$$\lim_{x \to 1^+} \frac{x-1}{x^3-1} \qquad \frac{0}{0}$$

FACTOR OUT: $x^3 - 1$

$$\lim_{x \to 1^+} \frac{x-1}{x^3-1} = \lim_{x \to 1^+} \frac{(x-1)'}{(x^3-1)'} = \lim_{x \to 1^+} \frac{1}{3x^2}$$

$$= \frac{1}{3 \cdot 1^2} = \frac{1}{3}$$

$a + 3 = \frac{1}{3} = b \qquad a = -\frac{8}{3}$

$b = \frac{1}{3}$

$$\lim_{x\to a}[f(x)+g(x)]=\lim_{x\to a}f(x)+\lim_{x\to a}g(x)$$

$$\lim_{x\to a}(f(x)\cdot g(x))=\lim_{x\to a}f(x)\cdot\lim_{x\to a}g(x)$$

$$\lim_{x\to a}f(g(x))=f(\lim_{x\to a}g(x))$$

ASSUMING f CONTINUOUS

<u>EX</u>: IF $f(3)=5$, $g(3)=-2$ f, g CONTINUOUS

WHAT IS $\displaystyle\lim_{x\to 3}\cos\left(\frac{f(x)+g(x)}{1-f(x)\cdot g(x)}\right)$

BY PROPERTIES OF LIMITS, PLUG IN $x=3$

$$\lim_{x\to 3}\cos\frac{f(x)+g(x)}{1-f(x)\cdot g(x)}=\cos\frac{f(3)+g(3)}{1-f(3)\cdot g(3)}$$

$$=\cos\frac{5-2}{1-5\cdot(-2)}=\cos\frac{3}{11}$$

EX: ASSUME x, [redacted] >0, $x-y=8$. FIND x, y SO THAT x^3-3y IS AS SMALL AS POSSIBLE. WHAT IS THE SMALLEST POSSIBLE VALUE?

USE CONSTRAINT $x-y=8$ TO EXPRESS x^3-3y AS A FUNCTION OF ONE VARIABLE

$Q(x)=x^3-3(x-8)$, $Q(x)=x^3-3x+24$

$x-y=8$, $y=x-8$

FIND THE MINIMUM OF $Q(x)$ FOR $0<x<+\infty$

FIND CRITICAL POINTS IN $(0,+\infty)$

$Q'(x)=3x^2-3$

FIND $0<c<+\infty$ S.T. $Q'(c)=0$

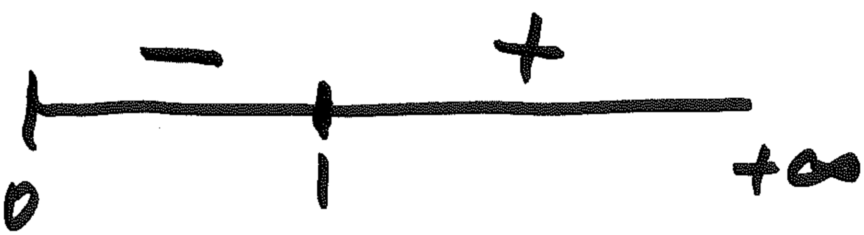
$3c^2-3=0$, $c=\pm 1$

$c=1$ ONLY CRITICAL POINT

$x=1$, $y=1-8=-7$, $x^3-3y=1-3\cdot(-7)=\underbrace{22}_{\text{MIN. VALUE}}$

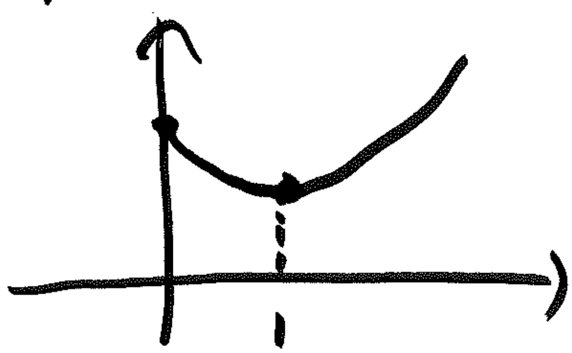
USE SOME TEST TO SHOW THAT c IS A MINIMUM

GLOBAL EXTREMUM TEST: LOOK AT THE SIGN OF Q'



SINCE $Q'(x) < 0$ ON $(0,1)$ AND $Q'(x) > 0$ ON $(1,+\infty)$, $c=1$ IS A <u>GLOBAL</u> MINIMUM

Q DECREASING ON $(0,1)$ AND INCREASING ON $(1,+\infty)$



SECOND DERIVATIVE TEST : $Q''(x) = 6x$

$Q''(1) = 6 > 0$

$c=1$ IS A LOCAL MINIMUM

TANGENT LINE

FIND THE TANGENT LINE TO THE GRAPH OF $y^2 - x^3 = 1$ AT $(1, \sqrt{2})$

TANGENT LINE: $y - \sqrt{2} = m(x-1)$

$m = \frac{dy}{dx}$ EVALUATED AT $(1, \sqrt{2})$

1) SOLVE FOR y $\quad y = \pm\sqrt{1+x^3}$

SINCE $y(1) > 0$, WE MUST HAVE

$$y(x) = \sqrt{1+x^3}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1+x^3}} \cdot (1+x^3)' = \frac{3x^2}{2\sqrt{1+x^3}}$$

WHEN $x=1$: $\frac{dy}{dx}(1) = \frac{3}{2\sqrt{2}}$

2) IMPLICIT DIFF.

$$\frac{d}{dx}(y^2 - x^3) = \frac{d}{dx} 1$$

$$\frac{d}{dx} y^2 = 2y \frac{dy}{dx}$$

$$\frac{d}{dx}(-x^3) = -3x^2$$

$$2y \frac{dy}{dx} - 3x^2 = 0$$

$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

EVALUATE AT $(1, \sqrt{2})$

$$\cancel{\frac{dy}{dx}} \quad m = \frac{3}{2\sqrt{2}}$$

$$y - \sqrt{2} = \frac{3}{2\sqrt{2}}(x-1)$$

$L(x) = \frac{3}{2\sqrt{2}}(x-1) + \sqrt{2}$ NEAR $x = 1$