REVIEW 3

M 3-4:30 PM OH

OPTIMIZATION

EX: $x, y > 0$ SO THAT $xy = 4$.
WHAT IS THE MINIMUM OF $2x + 3y$?

$xy = 4$, $y = \frac{4}{x}$

$Q(x) = 2x + 3 \cdot \frac{4}{x} = 2x + \frac{12}{x}$

DOMAIN OF Q: $(0, +\infty)$

FIND CRITICAL POINTS OF Q ON $(0, +\infty)$

$Q'(x) = 2 - \frac{12}{x^2}$

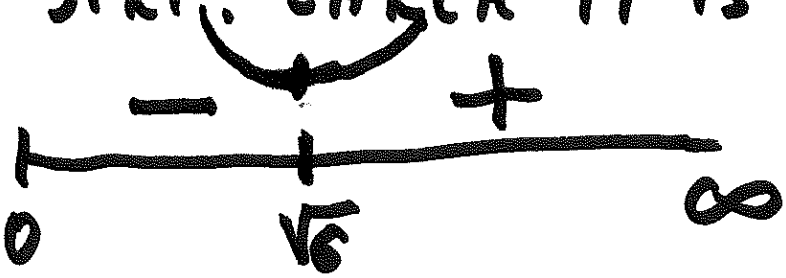
$\left(\frac{12}{x}\right)' = 12\left(\frac{1}{x}\right)' = 12 \cdot (x^{-1})' = 12 \cdot (-1) \cdot x^{-2}$

$Q'(c) = 0$ $\quad 2 - \frac{12}{c^2} = 0 \quad 2 = \frac{12}{c^2}$

$2c^2 = 12$, $c^2 = 6$, $c = \sqrt{6}$

$$\text{MINIMUM} = Q(\sqrt{6}) = 2\sqrt{6} + \frac{12}{\sqrt{6}} = 2\sqrt{6} + 2\sqrt{6}$$
$$= 4\sqrt{6}$$

LAST STEP: CHECK IT IS A GLOBAL MINIMUM



SIGN OF $Q'(x)$ $\quad$ $0 < x < \sqrt{6}$ $\quad$ $-$

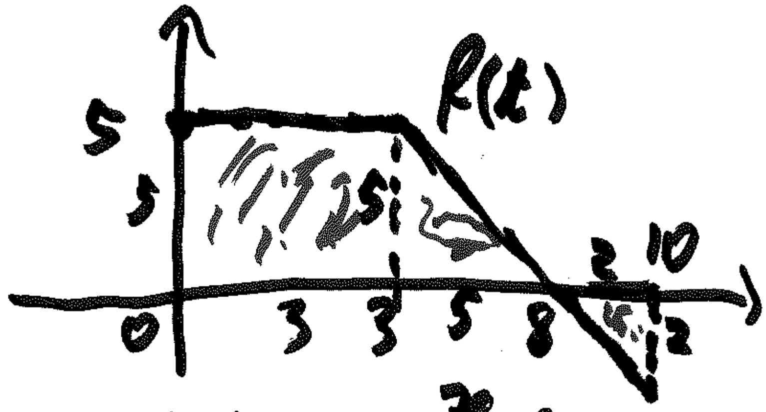
$Q'(1) = 2 - 12 = -10 < 0$

~~SIGN OF~~ $\quad$ $\sqrt{6} < x$ $\quad$ $+$

$Q'(5) = 2 - \frac{12}{25} > 0$

SO $\sqrt{6}$ IS A <u>GLOBAL</u> MINIMUM

<u>EX</u>: LET $f(t)$ BE AS BELOW



LOOK AT $F(x) = \int_0^x f(t)\,dt$

BY FTC I, $F'(x) = f(x)$

Q1: WHAT IS $F(10)$?

Q2: WHERE IS F DECREASING?

Q2 : F DECREASING WHEN $f<0$

$(8,10)$

Q1: BY ~~FTC II~~ DEF., $F(10) = \int_0^{10} f(t)\,dt$

$= \int_0^3 f(t)\,dt + \int_3^8 f(t)\,dt + \int_8^{10} f(t)\,dt$

$= 5\cdot 3 + \frac{5\cdot 5}{2} - \frac{2\cdot 2}{2} = 15 + \frac{21}{2}$

EX: $\int \frac{x^2e^x + 2xe^x}{x^2e^x+1}\,dx = \int \frac{du}{u}$

$= \ln|u| = \ln(x^2e^x+1) + C$

LET $u = x^2e^x+1$

$du = (x^2e^x+1)'\,dx = (2xe^x + x^2e^x)\,dx$

$(x^2e^x+1)' = (x^2e^x)' + (1)' = (x^2)'e^x + x^2(e^x)'$

$= 2xe^x + x^2e^x$

EX: $\int_0^5 \frac{1}{\sqrt{x+4}}\,dx = \int_4^9 \frac{1}{\sqrt{u}}\,du \overset{(*)}{=}$

NATURAL SUBSTITUTIONS:

$\sqrt{x+4} = u$ DOES NOT WORK

$x+4 = u$, $du = (x+4)'dx = dx$

$\overset{(*)}{=} \int_4^9 u^{-\frac{1}{2}}\,du = 2u^{\frac{1}{2}}\Big|_4^9 = 2\cdot 9^{\frac{1}{2}} - 2\cdot 4^{\frac{1}{2}}$

$= 2\cdot 3 - 2\cdot 2 = 2$

EX: NET CHANGE THEOREM

TOTAL NET CHANGE = INTEGRAL OF THE RATE OF CHANGE

ASSUME WATER FLOWS INTO A TANK AT THE RATE $f(t) = t^2 + 7t$ GALLONS/MIN. SAY THE WATER STARTS FLOWING WHEN $t=3$. HOW MANY GALLONS ARE IN THE TANK ~~AFTER~~ WHEN $t=8$?

$F(x)$ = AMOUNT OF WATER AFTER x MINUTES

$0 \leq x \leq 3 \quad F(x) = 0$

$F(x)$ [redacted] $F(3) = \int_3^x f(t)\,dt$

$x = 8, \ F(3) = 0$

$$F(8) = \int_3^8 t^2 + 7t \, dt = \frac{t^3}{3} + \frac{7}{2} t^2 \Big|_3^8$$

$$= \frac{8^3}{3} + \frac{7}{2} \cdot 8^2 - \left(\frac{3^3}{3} + \frac{7}{2} 3^2 \right) = \ldots$$

RATE OF CHANGE = DERIVATIVE

<u>EX</u>: $f(x) = x \ln(x^2+1)$

FIND THE LINEARIZATION OF f NEAR $x = 1$

$L(x) = f(1) + f'(1)(x-1)$

$f(1) = 1 \ln(1^2+1) = \ln 2$

$f'(x) = \left[x \ln(x^2+1) \right]' = x' \ln(x^2+1) + x \left(\ln(x^2+1) \right)'$

$x' = 1$

$$[\ln(x^2+1)]' = (\ln')(x^2+1) \cdot (x^2+1)'$$

$$= \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$$

$$f'(x) = \ln(x^2+1) + \frac{2x^2}{x^2+1}$$

$$x=1: \ f'(1) = \ln 2 + \frac{2}{2} = \ln 2 + 1$$

$$L(x) = \ln 2 + (\ln 2+1)(x-1) = (\ln 2+1)x - 1$$