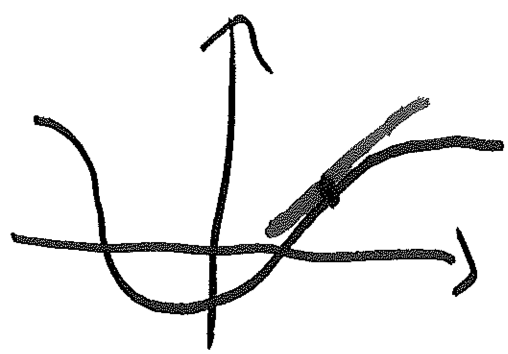
LIMITS AND DERIVATIVES

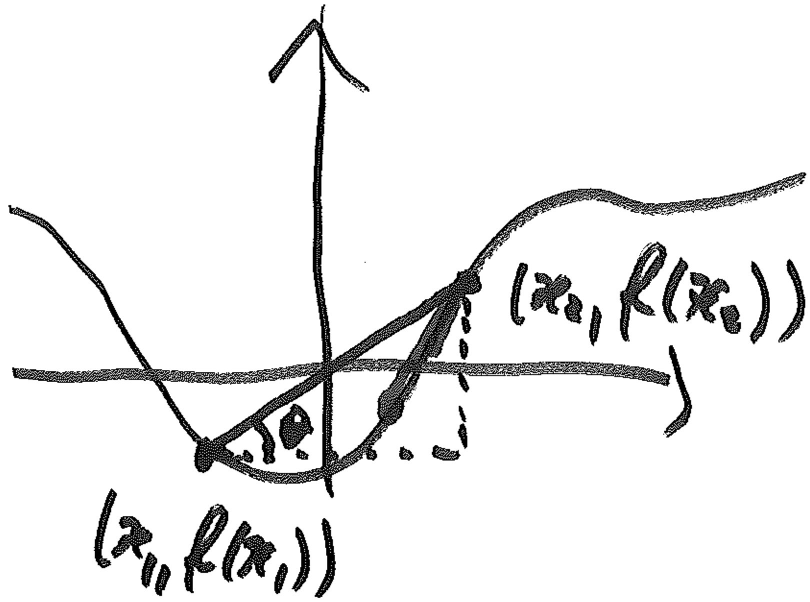
TANGENT AND VELOCITY PROBLEMS

Q1: GIVEN A FUNCTION $y=f(x)$, AND SOME POINT $(x_0, f(x_0))$, WHAT IS THE TANGENT LINE THROUGH $(x_0, f(x_0))$?



Q2: ASSUMING WE KNOW THE POSITION OF AN OBJECT AT ALL TIMES, CAN WE DETERMINE HOW FAST IT MOVED AT EACH TIME? INSTANTANEOUS VELOCITY

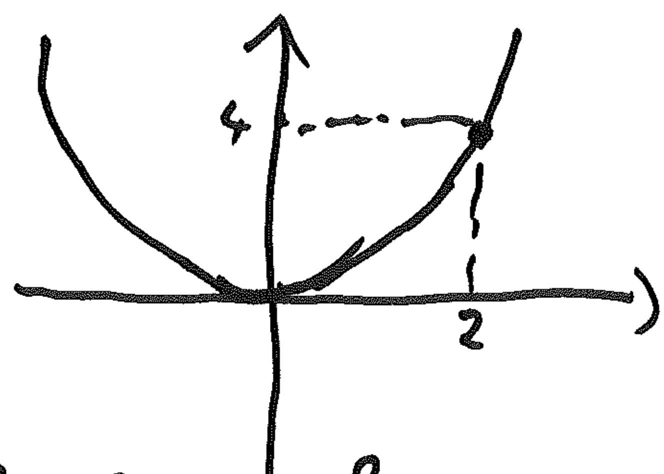
GIVEN TWO POINTS $(x_1, f(x_1))$ AND $(x_2, f(x_2))$, WHAT IS THE SLOPE OF THE SECANT?



$$\text{SLOPE} = \tan\theta = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

IF $x_1 = x_2$ $\quad \dfrac{f(x_1) - f(x_1)}{x_1 - x_1}$ DIVIDE BY 0 X

<u>EX</u>: $f(x) = x^2$, LOOK AT $(2,4)$



PICK $x_1 = 2$, $x_2 = 2 + h$

SLOPE OF SECANT: $\dfrac{(2+h)^2 - 4}{h}$

h	SLOPE
1	5
0.1	4.1
-0.1	3.9
0.01	4.01
-0.01	3.99
⋮	

4

$$\frac{(2+h)^2-4}{h} = \frac{\cancel{4}+4h+h^2-\cancel{4}}{h} = \frac{\cancel{h}(4+h)}{\cancel{h}} = 4+h$$

<u>Q2</u> ASSUME THAT A BIKER'S POSITION IS GIVEN BY $y(t)=t^2$ AT ALL TIMES

I) WHAT IS THE AVERAGE VELOCITY BETWEEN TIME 1 AND TIME 3 ?

$$\text{AVE. VEL.} = \frac{\text{DIST.}}{\text{TIME}} = \frac{3^2-1^2}{3-1} = 4$$

II) INSTANTANEOUS VELOCITY : LIMIT OF AVERAGE VELOCITIES

BOTH THE SLOPE OF THE TANGENT LINE, AND THE INSTANTANEOUS VELOCITY ARE EXAMPLES OF <u>LIMITS</u>

"DEF": THE LIMIT OF A FUNCTION $f(x)$ AS x APPROACHES a, DENOTED BY $\lim_{x \to a} f(x)$, IS THE VALUE $f(x)$ "GETS CLOSE TO" AS x "GETS CLOSE TO" a (BUT NOT a)

<u>REMARK 1</u>: f MUST BE DEFINED ON AN OPEN INTERVAL CONTAINING a, BUT NOT NECESSARILY AT a.

<u>REMARK 2</u>: THE LIMIT DOES NOT ALWAYS EXIST

EX 1: $f(x)=3$

$$\lim_{x\to 5} f(x)=3$$

EX 2: $f(x)=x$

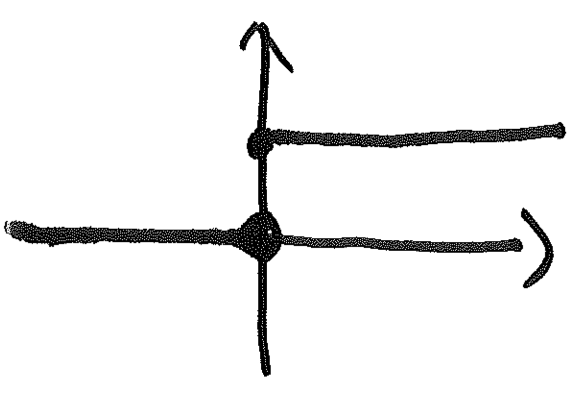
$$\lim_{x\to 10} f(x)=10$$

EX 3: $f(x)=\frac{x^2}{x}$

DOMAIN : $x\neq 0$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0} x=0$$

EX 4: $H(x)=\begin{cases} 0, & x<0 \\ 1, & x\geq 0 \end{cases}$



$$\lim_{x\to 0} H(x) \text{ DNE}$$