LAST TIME: LIMITS

$$H(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

$\lim_{x \to 0} H(x)$ DNE

AS x APPROACHES 0, $x < 0$, $H(x)$ APPROACHES 0

AS x APPROACHES 0, $x > 0$, $H(x)$ APPROACHES 1

ONE-SIDED LIMITS

$\lim_{x \to a^-} f(x)$ IS THE NUMBER THAT f APPROACHES AS x "GETS CLOSE" TO a, $x < a$ (LEFT SIDE LIMIT)

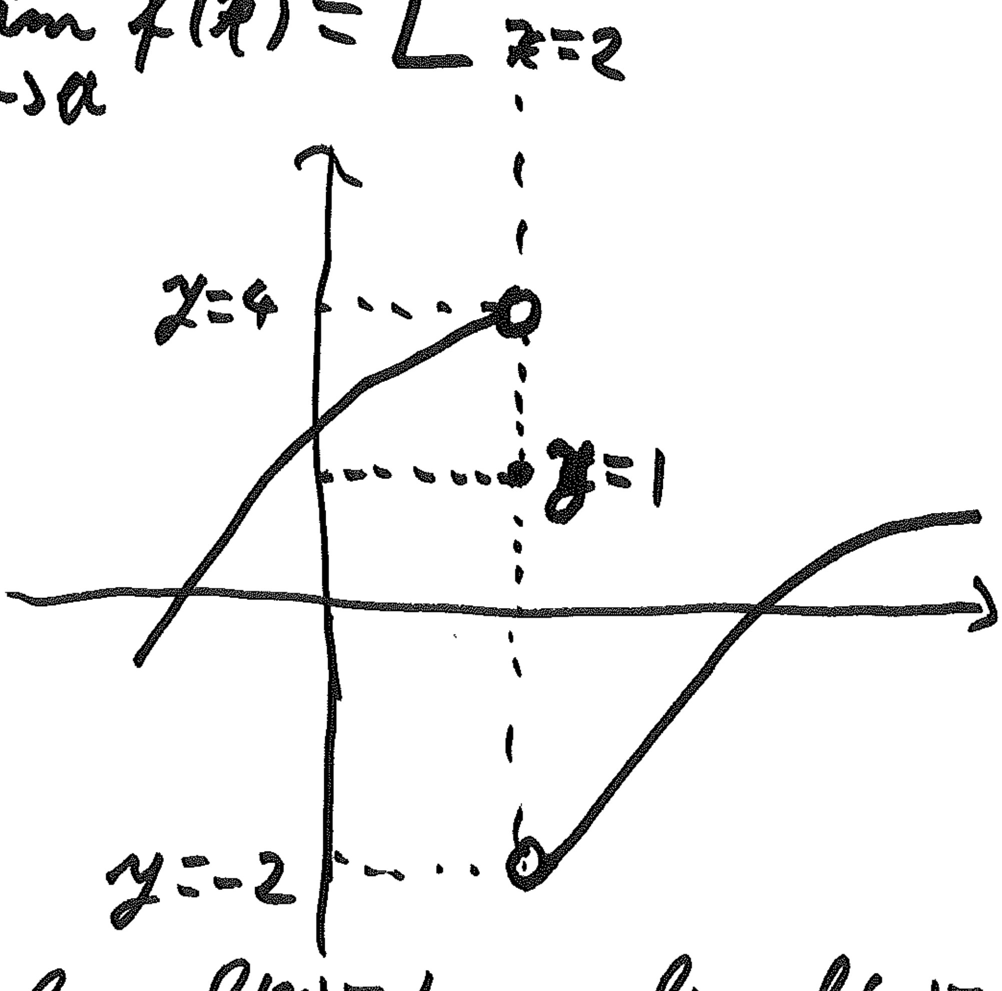
$\lim_{x \to a^+} f(x)$ IS THE NUMBER THAT

f APPROACHES AS x "GETS CLOSE"

TO a, $x > a$ (RIGHT SIDE LIMIT)

IF $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$, THEN

$\lim_{x \to a} f(x) = L$

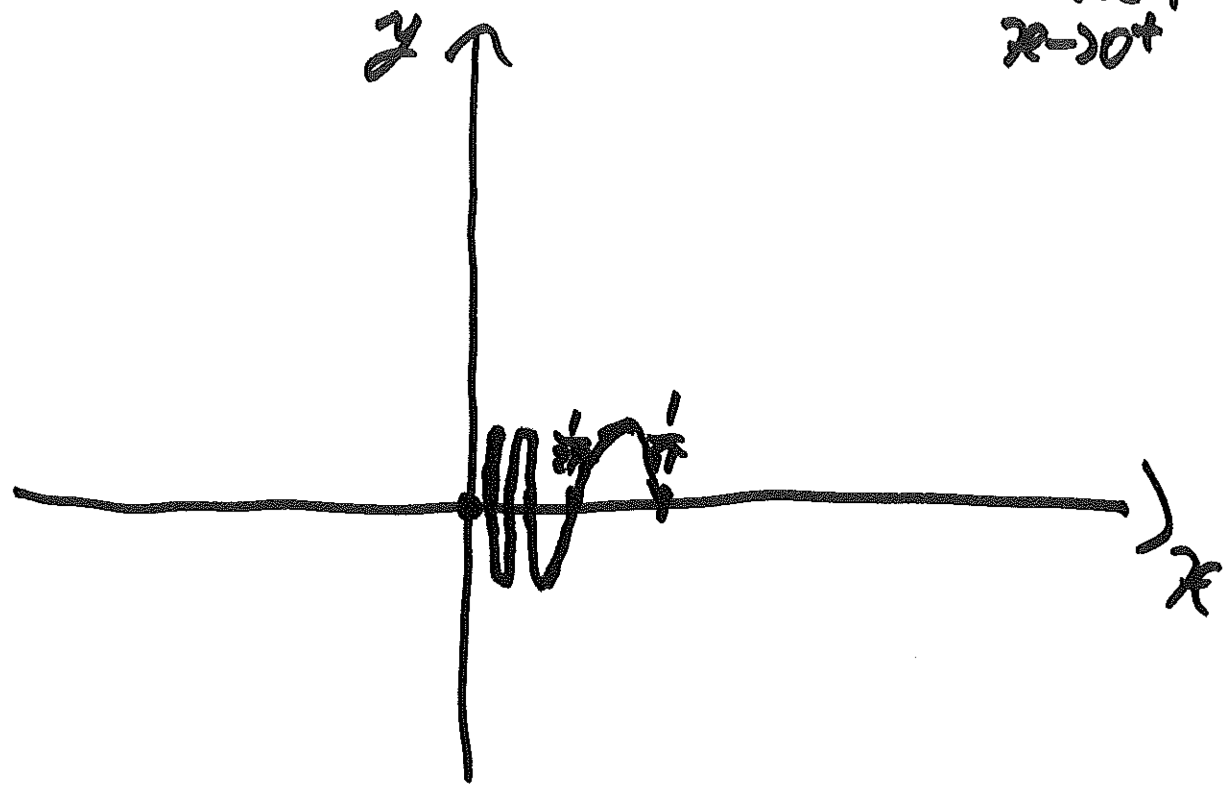


$\lim_{x \to 2^-} f(x) = 4$ $\lim_{x \to 2^+} f(x) = -2$

$f(2) = 1$

Ex: $f(x) = \begin{cases} \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$\lim_{x \to 0^+} f(x)$ DNE



$\sin\frac{1}{x} = 0 \qquad \frac{1}{x} = k\pi, \quad x = \frac{1}{k\pi}$

$\frac{1}{\pi}, \frac{1}{2\pi}, \frac{1}{3\pi}, \frac{1}{4\pi}, \ldots$

INFINITE LIMITS

"DEF": f DEFINED IN AN OPEN INTERVAL AROUND a (NOT NECESSARILY AT a).

THEN $\lim_{x \to a} f(x) = +\infty$ IF THE VALUES $f(x)$ BECOME "ARBITRARILY LARGE" AS x "GETS CLOSE TO" a

SIMILARLY $\lim_{x \to a} f(x) = -\infty$

$$\lim_{x \to a^-} f(x) = +\infty, \quad \lim_{x \to a^+} f(x) = +\infty$$

$$\lim_{x \to a^-} f(x) = -\infty, \quad \lim_{x \to a^+} f(x) = -\infty$$

<u>EX</u>: $\lim_{x \to 0} \frac{1}{x^2} = +\infty$, $\lim_{x \to 0} \frac{-5}{x^2} = -\infty$

$$\lim_{x \to 0} \frac{1}{x} = \text{DNE} \qquad \lim_{x \to 0^+} \frac{1}{x} = +\infty$$

$$\lim_{x \to 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x \to 5} \frac{1}{(x-5)^n}$$

$n = 1, 2, 3, 4, \ldots$

$$\lim_{x \to 5^+} \frac{1}{x-5} = +\infty$$

$$\lim_{x \to 5^-} \frac{1}{x-5} = -\infty$$

$$\lim_{x \to 5} \frac{1}{(x-5)^2} = +\infty$$

FOR ODD POWERS: $\displaystyle\lim_{x \to 5^+} \frac{1}{(x-5)^n} = +\infty$

$$\lim_{x \to 5^-} \frac{1}{(x-5)^n} = -\infty$$

FOR EVEN POWERS: $\displaystyle\lim_{x \to 5^+} \frac{1}{(x-5)^n} = \lim_{x \to 5^-} \frac{1}{(x-5)^n}$

$$= +\infty$$

VERTICAL ASYMPTOTES

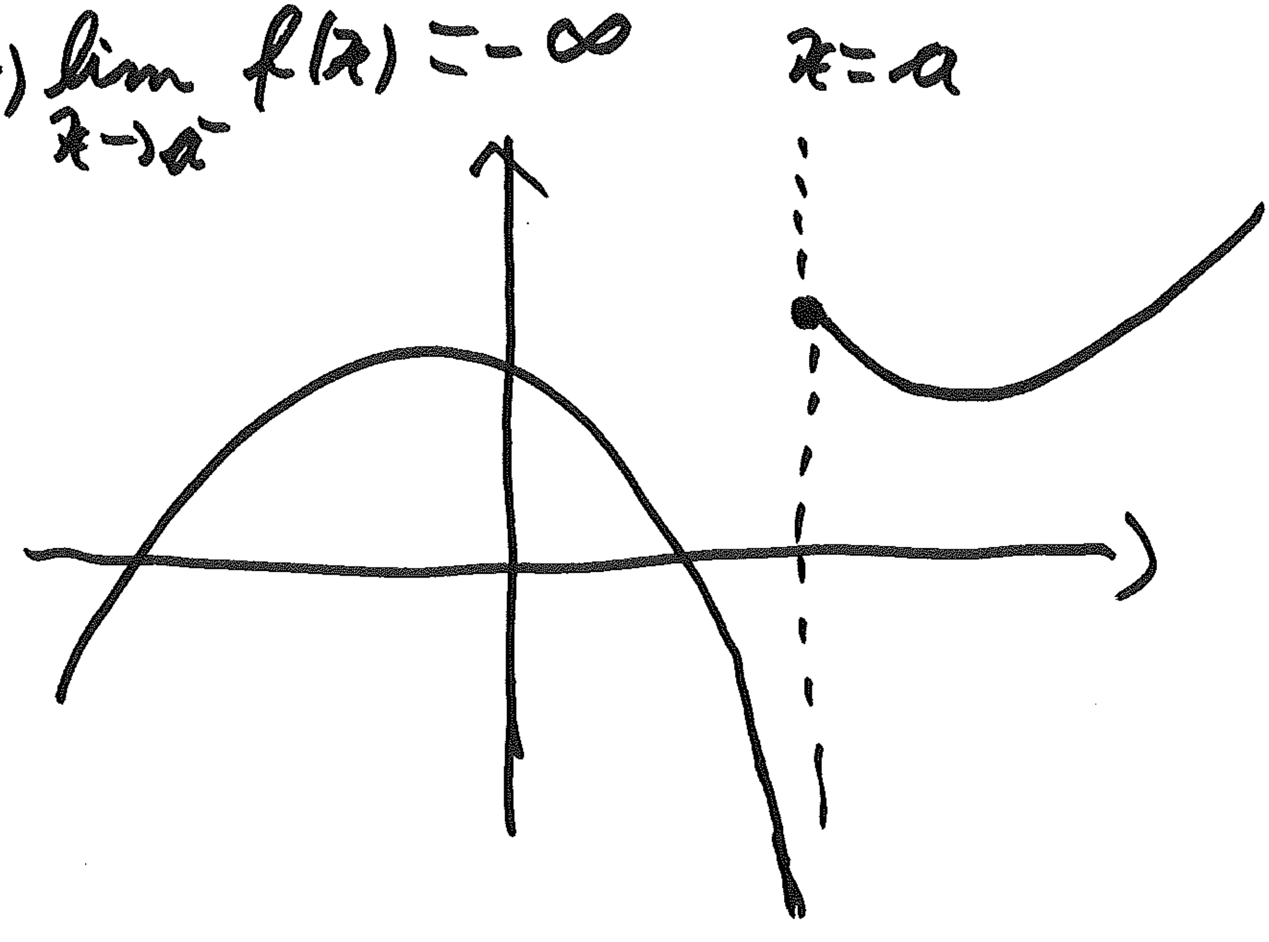
GIVEN A FUNCTION $f(x)$, WE SAY THAT $x=a$ IS A <u>VERTICAL ASYMPTOTE</u> IF EITHER:

i) $\lim_{x \to a^+} f(x) = +\infty$, OR

ii) $\lim_{x \to a^+} f(x) = -\infty$, OR

iii) $\lim_{x \to a^-} f(x) = +\infty$, OR

iv) $\lim_{x \to a^-} f(x) = -\infty$



IN PRACTICE, LOOK AT WHERE THE DENOMINATOR BECOMES 0

EX: $f(x) = \dfrac{x-5}{x(x^2-16)}$

$x(x^2-16) = x(x-4)(x+4)$

DOMAIN: $x \neq 0, 4, -4$

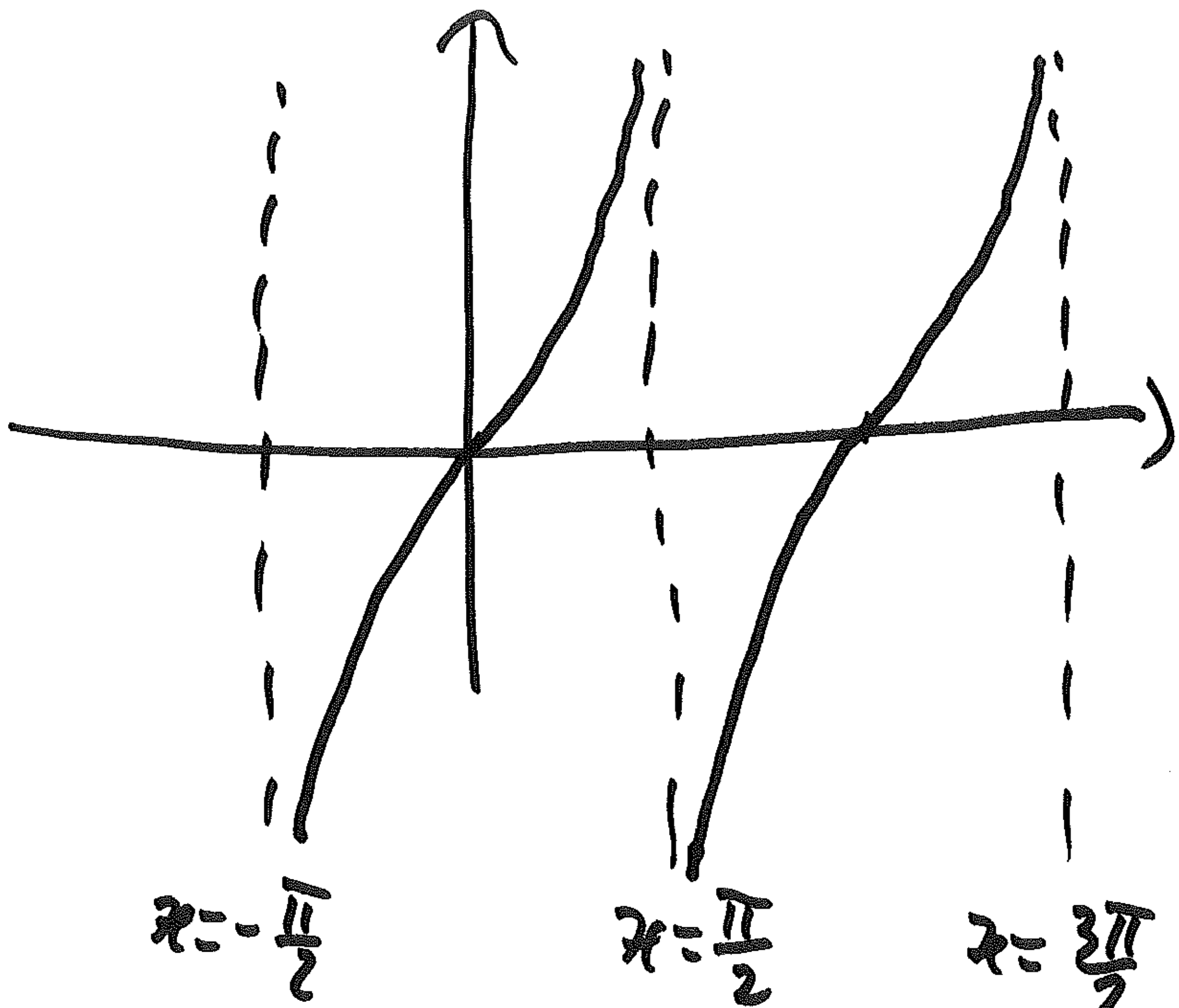
GRAPH, SEE THAT THERE ARE VERTICAL ASYMPTOTES AT $x=0$, $x=-4$, $x=4$

CAREFUL: $f(x) = \dfrac{x-4}{x(x^2-16)}$

$$f(x) = \frac{\cancel{x-4}}{x\cancel{(x-4)}(x+4)} = \frac{1}{x(x+4)} \qquad \text{"}\frac{0}{0}\text{"}$$

EX: $f(x) = \tan x = \dfrac{\sin x}{\cos x}$

$\cos x = 0 \quad x = \frac{\pi}{2} + k\pi$

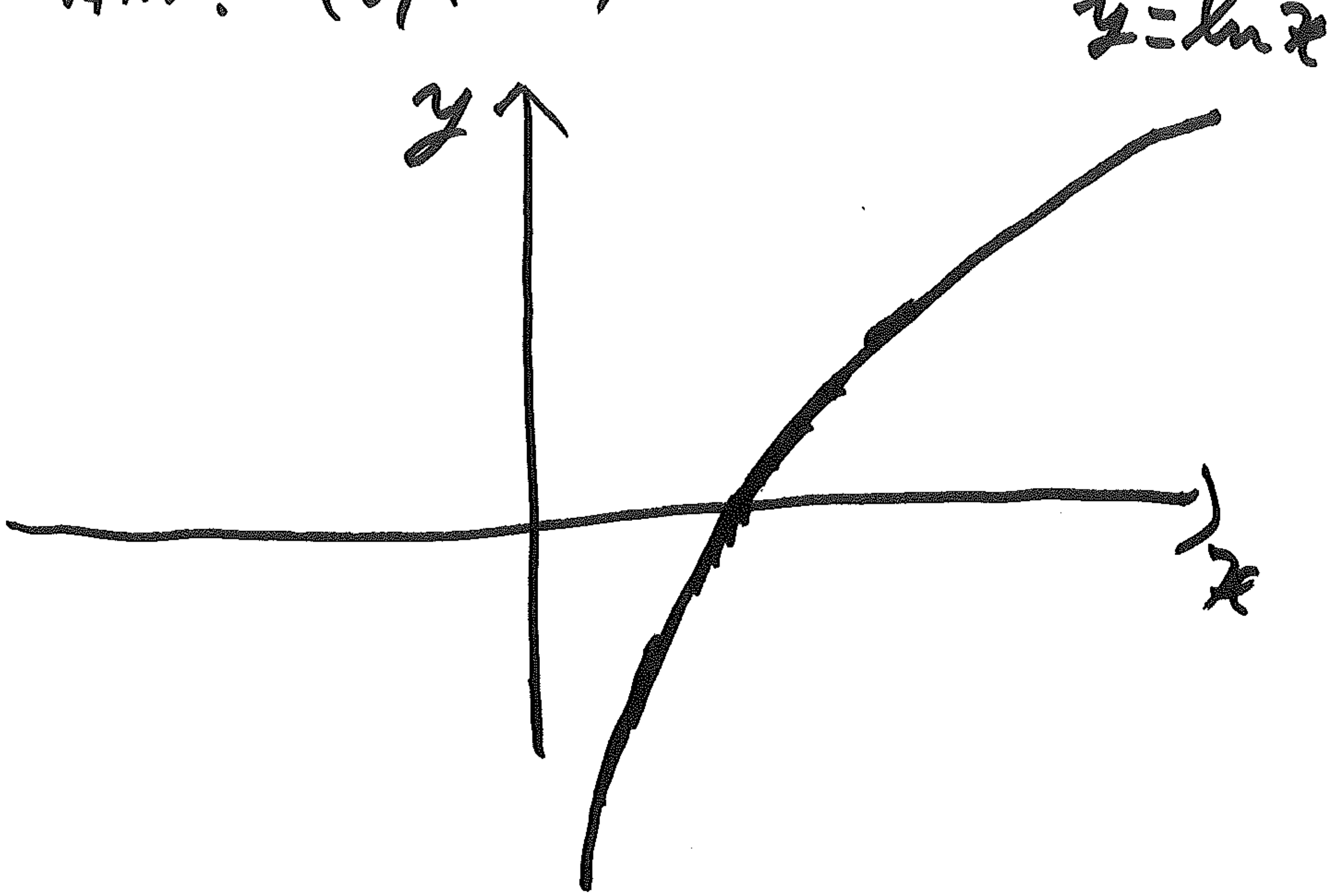


$$\lim_{x \to \frac{\pi}{2}^-} f(x) = +\infty$$

$$\lim_{x \to \frac{\pi}{2}^+} f(x) = -\infty$$

EX: $f(x) = \ln x$

DOMAIN: $(0, +\infty)$



$$\lim_{x \to 0^+} \ln x = -\infty$$