REVIEW SESSION

1) FUNCTIONS: DOMAIN, RANGE, INVERSE, COMPOSITION

2) EXPONENTIAL / LOGARITHMS

3) TRIGONOMETRIC / INVERSE TRIG. FNS

4) TANGENT / VELOCITY PROBLEMS

5) LIMITS

Ex 1 $f(x) = \frac{x}{2x-5}$

i) WHAT IS THE DOMAIN?

ii) WHAT IS THE RANGE?

iii) WHAT IS THE INVERSE?

iv) COMPUTE $f \circ f$

v) FIND A FUNCTION g SO THAT

$$(g \circ f)(x) = \underbrace{x+1}_{h(x)}$$

i) DENOMINATOR $\neq 0$

$2x-5 \neq 0$, $x \neq \frac{5}{2}$

ii) + iii) : SOLVE $f(x)=y$

$$\frac{x}{2x-5} = y \quad | \cdot (2x-5)$$

$$x = y(2x-5)$$

$$x = 2xy - 5y \quad | -2xy$$

$$x - 2xy = -5y$$

$$x(1-2y) = -5y$$

$$x = \frac{-5y}{1-2y} = \frac{5y}{2y-1}$$

RANGE: $y \neq \frac{1}{2}$

iii) $f^{-1} : \{x \neq \frac{1}{2}\} \rightarrow \{x \neq \frac{5}{2}\}$

$$f^{-1}(x) = \frac{5x}{2x-1}$$

iv) $(f\circ f)(x)=f(f(x))$

$$=\frac{f(x)}{2f(x)-5}=\frac{\frac{x}{2x-5}}{2\cdot\frac{x}{2x-5}-5}=\frac{x}{2x-5(2x-5)}$$

$$=\frac{x}{-8x+25}$$

v) $g(f(x))=h(x)$

WRITE $x=f^{-1}(y)$

$g(f(x))=g(f(f^{-1}(y)))=g(y)$

$h(x)=h(f^{-1}(y))$

$g(y)=h(f^{-1}(y))$

$g=h\circ f^{-1}$

IN OUR CASE: $h(x)=x+1$

$f^{-1}(x)=\frac{5x}{2x-1}$

$g(x)=(h\circ f^{-1})(x)=f^{-1}(x)+1=\frac{5x}{2x-1}+1$

EX 2 : EXP/LOG

$$a^{x+y} = a^x \cdot a^y$$

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b(x^n) = n \log_b x$$

SOLVE $2^{x^2} = \frac{16^x}{16}$

$$\frac{16^x}{16} = 16^x \cdot 16^{-1} = 16^{x-1}$$

$$2^{x^2} = 16^{x-1}$$

$$\log_2(2^{x^2}) = \log_2(16^{x-1})$$

$$x^2 \log_2 2 = (x-1)\log_2 16$$

$$x^2 = 4(x-1)$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)^2 = 0 \qquad x = 2$$

EX 3 SIMPLIFY

$$\sec(2\arcsin x) = \frac{1}{\cos(2\sin^{-1}x)}$$

$$\arcsin x = \sin^{-1}x$$

$$\sin^{-1}x = y, \quad x = \sin y$$

$$\frac{1}{\cos(2y)} = \frac{1}{\cos^2 y - \sin^2 y} = \frac{1}{1-2\sin^2 y}$$

$$\cos^2 y = 1 - \sin^2 y$$

$$\sec(2\arcsin x) = \frac{1}{1-2x^2}$$

LOOK AT $\sin^{-1}(\sin x)$

RANGE OF $\sin^{-1}$ IS $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

IF $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, $\sin^{-1}(\sin x) = x$

IF $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$, $\sin^{-1}(\sin x) = \sin^{-1}(\sin(\pi - x))$

$= \pi - x$

CHECK: $\sin x = \sin(\pi - x)$

$$\sin^{-1}\left(\sin\frac{11\pi}{4}\right) = \sin^{-1}\left(\sin\frac{3\pi}{4}\right) = \pi - \frac{3\pi}{4}$$

$$= \frac{\pi}{4}$$

$$\sin(x+2\pi) = \sin x$$

$$\frac{\pi}{2} < \frac{3\pi}{4} < \frac{3\pi}{2}$$

$$\frac{11\pi}{4} = 2\pi + \frac{3\pi}{4}$$