REVIEW 2

1) VELOCITY/AVERAGE VELOCITY

   INSTANTANEOUS

2) LIMITS

<u>EX 1</u>: $f(t) = 2t^2 + t$

i) AVERAGE OVER $[4, 4.001]$

ii) AVERAGE OVER $[3.999, 4]$

iii) INSTANTANEOUS VELOCITY WHEN $t = 4$

AVERAGE OF f OVER $[a, b]$ $\quad \dfrac{f(b) - f(a)}{b - a}$

i) $$\frac{f(4.001) - f(4)}{4.001 - 4} = \frac{2 \cdot 4.001^2 + 4.001 - 2 \cdot 4^2 - 4}{.001}$$

$$= 17.002$$

ii) $$\frac{f(4) - f(3.999)}{4 - 3.999} = 16.998$$

iii) $\lim_{h \to 0} \frac{f(4+h)-f(4)}{h} = 17$

$$\frac{2(4+h)^2+(4+h)-2\cdot 4^2-4}{h}$$

$$= \frac{2(4^2+8h+h^2)+4+h-2\cdot 4^2-4}{h}$$

$$= \frac{\cancel{2\cdot 4^2}+16h+2h^2+\cancel{4}+h-\cancel{2\cdot 4^2}-\cancel{4}}{h}$$

$$= \frac{2h^2+17h}{h} = \frac{\cancel{h}(2h+17)}{\cancel{h}} = 2h+17$$

$$\lim_{h \to 0}(2h+17) = 2\cdot 0+17 = 17$$

LIMITS

USUALLY $\lim_{x \to a} f(x) = f(a)$

EXCEPTIONS:

1) DIVIDING BY ZERO
2) FUNCTION NOT DEFINED NEAR a
3) PIECEWISE FUNCTION AT a

EX: $\lim_{x \to 5} \frac{(x-5)^2}{x+5} = 0$

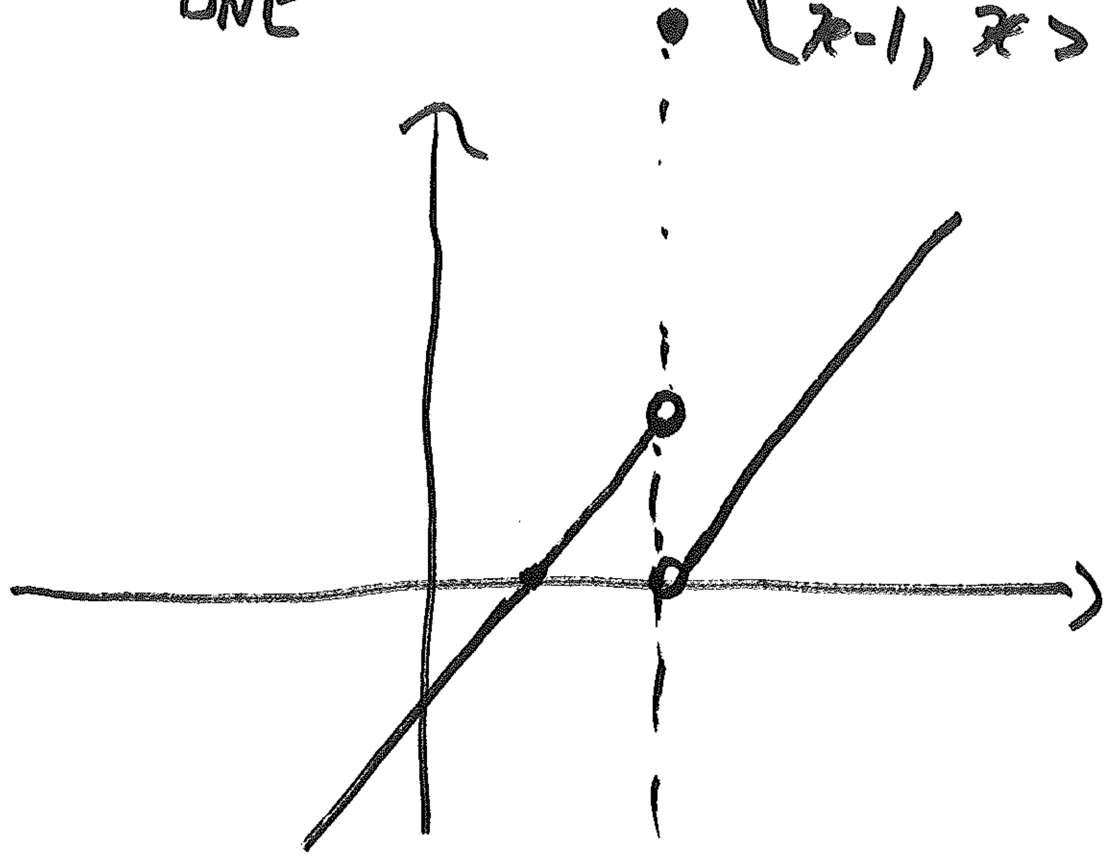
PLUG IN $x=5$: $\frac{(5-5)^2}{5+5} = \frac{0}{10} = 0$

$\lim_{x \to 2} \sqrt{x-2}$ DNE SINCE $\sqrt{x-2}$ DNE WHEN $x<2$

$\lim_{x \to 2^+} \sqrt{x-2} = \sqrt{2-2} = 0$

$$\lim_{x \to 1} f(x), \quad f(x) = \begin{cases} x+1, & x<1 \\ 42, & x=1 \\ x-1, & x>1 \end{cases}$$

DNE



$x=1$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x+1 = 2$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x-1 = 0$$

$$\lim_{x \to a} \frac{g(x)}{h(x)} \quad , \; h(a) = 0$$

IF $g(a) \neq 0$, THEN $\lim_{x \to a} \frac{g(x)}{h(x)}$

$\to$ DNE

$\to +\infty$

$\to -\infty$

$$\lim_{x \to 3} \frac{x-5}{x-3} \quad \text{DNE}$$

PLUG IN $x=3$: $\frac{-2}{0}$??

$$\lim_{x \to 3^+} \frac{x-5}{x-3} \approx \frac{-2}{0^+} = -\infty$$

$$\lim_{x \to 3^-} \frac{x-5}{x-3} \approx \frac{-2}{0^-} = +\infty$$

$$\lim_{x \to 3} \frac{x-5}{(x-3)^4} = -\infty$$

IF $g(a)=0$ "$\frac{0}{0}$" CASE

FACTOR NUMERATOR/DENOMINATOR

EX: $\lim\limits_{x \to 1} \frac{x^4 - x^2}{x-1} = 2$

PLUG IN $x=1$: $\frac{1^4 - 1^2}{1-1} = \frac{0}{0}$??

$$\frac{x^4 - x^2}{x-1} = \frac{x^2(x^2-1)}{x-1} = \frac{x^2\cancel{(x-1)}(x+1)}{\cancel{x-1}}$$

$$= x^2(x+1)$$

$$\lim_{x \to 1} x^2(x+1) = 1^2 \cdot (1+1) = 2$$

THM 1: IF $f(x) \le g(x)$

THEN $\lim\limits_{x \to a} f(x) \le \lim\limits_{x \to a} g(x)$

THM 2: IF $f(x) \le g(x) \le h(x)$, $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} h(x) = L$

THEN $\lim\limits_{x \to a} g(x) = L$

EX: $g(x) = x^4 \sin\frac{1}{x}$

$$\lim_{x \to 0} g(x)$$

NAIVE TRY: $\lim_{x \to 0} g(x) = \lim_{x \to 0} x^4 \cdot \lim_{x \to 0} \sin\frac{1}{x}$

$= 0 \cdot \text{DNE}$??

$-1 \leq \sin\frac{1}{x} \leq 1 \quad | \cdot x^4$

$$\underbrace{-x^4}_{f(x)} \leq x^4 \sin\frac{1}{x} \leq \underbrace{x^4}_{h(x)}$$

$\lim_{x \to 0} f(x) = 0$ $\qquad \lim_{x \to 0} g(x) = 0$

$\lim_{x \to 0} h(x) = 0$

EX: $f(x), g(x)$ $\quad \lim_{x \to 0} f(x) = 5$, $\lim_{x \to 0} g(x) = -1$

WHAT IS $\lim_{x \to 0} \left(2f(x) - 3g^2(x)\right) \xleftarrow{\text{LIMIT LAWS}}$

$2 \cdot 5 - 3 \cdot (-1)^2 = 7$