DERIVATIVE

INTEGRAL

$F(x)$ $F'(x)$ $\int F'(x)\,dx = F(x) + C$

TECHNIQUES FOR INTEGRATION

INTEGRATION BY PARTS

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

$$\int [f(x)g(x)]'\,dx = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

$$f(x)g(x) = \underbrace{\boxed{\int f'(x)g(x)\,dx}}_{\text{hard}} + \underbrace{\boxed{\int f(x)g'(x)\,dx}}_{\text{easy}}$$

$$\boxed{\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx}$$

IBP

$$\boxed{\int g\,df = fg - \int f\,dg}$$

LEIBNITZ NOTATION

$$\frac{df}{dx} = f'(x), \quad df = f'(x)\,dx$$

EX.1: $\int x\cos x\,dx = \int \overset{f_n}{x}\,\overset{g}{(\sin x)'}\,dx$

$\boxed{\int \cos x\,dx = \sin x}$ $= x\sin x - \int x'\sin x\,dx$

$= x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$

$\int \sin x\,dx = -\cos x + C$

EX.2: $\int x^2\cos x\,dx = \int x^2(\sin x)'dx = x^2\sin x$

$- \int (x^2)'\sin x\,dx = x^2\sin x - \int 2x\sin x\,dx$

HW: do $\int x\sin x\,dx$

EX3: $\int \ln x\,dx = \int 1\cdot\ln x\,dx = \int (x)'\ln x\,dx$

$= x\ln x - \int x(\ln x)'dx$

$= x\ln x - \int x\cdot\frac{1}{x}\,dx = x\ln x - \int 1\,dx$

$= x\ln x - x + C$

EX4: $\int e^x\sin x\,dx = \int (e^x)'\sin x\,dx$

$= e^x\sin x - \int e^x(\sin x)'dx$

$= e^x\sin x - \int e^x\cos x\,dx$

$= e^x\sin x - \int (e^x)'\cos x\,dx$

$= e^x\sin x - [e^x\cos x - \int e^x(\cos x)'dx]$

$= e^x\sin x - e^x\cos x + \int e^x(-\sin x)dx$

$\int e^x\sin x\,dx = e^x\sin x - e^x\cos x - \int e^x\sin x\,dx$

$$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x$$

$$\boxed{\int e^x \sin x\,dx = \frac{1}{2}\left[e^x \sin x - e^x \cos x\right]}$$

DEFINITE INTEGRALS

$$\int_a^b F(x)\,dx = \text{number}$$

$$\left[f(x)g(x)\right]' = f'(x)\,g(x) + f(x)\,g'(x)$$

$$\int_a^b \left[f(x)g(x)\right]'dx = \int_a^b f'(x)g(x)\,dx + \int_a^b f(x)g'(x)\,dx$$

$$\int_a^b F'(x)\,dx \overset{FTC}{=} F(b) - F(a) = F(x)\Big|_a^b$$

$$f(x)g(x)\Big|_a^b = \int_a^b f'(x)\,g(x)\,dx + \int_a^b f(x)\,g'(x)\,dx$$

$$\boxed{\int_a^b f'(x)\,g(x)\,dx = f(x)g(x)\Big|_a^b - \int_a^b f(x)\,g'(x)\,dx}$$

$\underline{EX}$: $\int_0^{\pi} x\cos x\,dx = \int_0^{\pi} x(\sin x)'\,dx$

$= x\sin x\Big|_0^{\pi} - \int_0^{\pi} \sin x\,dx$

$= \pi\sin\pi - 0\cdot\sin 0 - (-\cos x)\Big|_0^{\pi}$

$= 0 - 0 - [(-\cos\pi) - (-\cos 0)]$

$= -1 - 1 = -2$