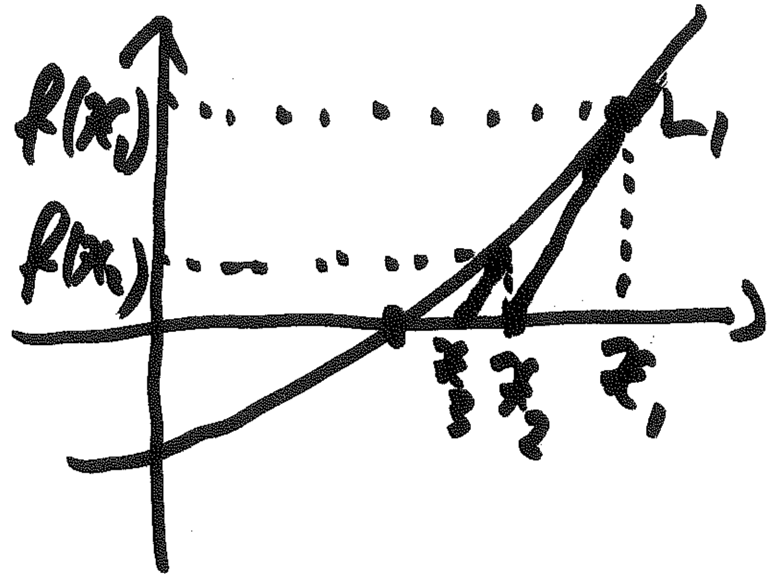
SEQUENCES

EX: NEWTON'S METHOD

$f(x) = 0$



$x_1, x_2, x_3 \ldots$

$\lim_{n \to \infty} x_n = r, \quad f(r) = 0$

EQUATION FOR L_1

$y - f(x_1) = f'(x_1)(x - x_1)$

$0 - f(x_1) = f'(x_1)(x_2 - x_1)$

$-\frac{f(x_1)}{f'(x_1)} = x_2 - x_1$

$$\boxed{x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

<u>EX</u>: $f(x) = x^2 - 2$

$f(x) = 0$, $x = \pm\sqrt{2}$ $\qquad \sqrt{2} \approx 1.4142\ldots$

$x_1 = 2$, $\quad x_{n+1} = x_n - \dfrac{x_n^2 - 2}{2x_n}$

$$x_2 = x_1 - \frac{x_1^2 - 2}{2x_1} = 2 - \frac{2^2 - 2}{2 \cdot 2} = \frac{3}{2}$$

$$x_3 = x_2 - \frac{x_2^2 - 2}{2x_2} = \frac{3}{2} - \frac{\left(\frac{3}{2}\right)^2 - 2}{2 \cdot \frac{3}{2}}$$

$$\approx 1.4166$$

$$\vdots$$

SERIES

$\{a_n\}$ SEQUENCE

$$s_n = a_1 + a_2 + \dots + a_n$$

$$\sum_{n=1}^{\infty} a_n = \sum a_n = \lim_{n \to \infty} s_n$$

IF THE LIMIT EXISTS, CONVERGENT $\to$ FINITE

IF NOT, DIVERGENT

<u>EX 1</u>: $a_n = 2^{-n}$

$$\sum_{n=1}^{\infty} a_n = ?$$

$$s_1 = a_1 = 2^{-1} = \frac{1}{2}$$

$$s_2 = a_1 + a_2 = 2^{-1} + 2^{-2} = \frac{3}{4}$$

$$s_3 = s_2 + a_3 = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$$

$$s_4 = \frac{15}{16} \quad \dots \quad s_n = \frac{2^n - 1}{2^n} \to 1$$

EX 2: $\sqrt{2} = 1.4142\ldots$

$$\sqrt{2} = 1 + \frac{4}{10} + \frac{1}{100} + \frac{4}{1000} + \frac{2}{10^4} + \ldots$$

EX 3: $a_n = 1$

$s_1 = 1$, $s_2 = s_1 + a_2 = 1 + 1 = 2$

$s_3 = 3$, $s_4 = 4$, ...

$s_n = n$, $\lim_{n\to\infty} s_n = \lim_{n\to\infty} n = \infty$

DIVERGENT

EX 4: $a_n = (-1)^n$

$a_1 = -1$, $a_2 = 1$, $a_3 = -1$, $a_4 = 1$, ..

$s_1 = -1$, $s_2 = -1 + 1 = 0$, $s_3 = -1$, $s_4 = 0$

$\lim_{n\to\infty} s_n$ DNE DIVERGENT

GEOMETRIC SERIES

PICK r TO BE SOME NUMBER ($\neq 0$)

$$a_n = r^{n-1}$$

$$a_1 = r^{1-1} = r^0 = 1$$

$$a_2 = r^{2-1} = r$$

$$a_3 = r^2 \quad \ldots$$

$$1 + r + r^2 + r^3 + \ldots = \sum_{n=1}^{\infty} r^{n-1}$$

$$s_n = 1 + \cancel{r} + \cancel{r^2} + \ldots + r^{n-1}$$

$$r s_n = \cancel{r} + \cancel{r^2} + r^3 + \ldots + r^n$$

SUBTRACT

$$s_n - r s_n = 1 - r^n$$

$$\boxed{s_n = \frac{1 - r^n}{1 - r}} \qquad (r \neq 1)$$

$$\lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$$

$$\lim_{n\to\infty} 2^n = +\infty$$

$$\lim_{n\to\infty} \left(-\frac{1}{2}\right)^n = 0$$

$$\lim_{n\to\infty} (-2)^n = DNE$$

$$\lim_{n\to\infty} r^n = \begin{cases} 0, \text{ if } |r|<1 \\ +\infty \text{ or } DNE, |r|>1 \\ 1, \text{ if } r=1 \end{cases}$$

$$\lim_{n\to\infty} s_n = \begin{cases} \dfrac{1}{1-r}, & |r|<1 \\ +\infty \text{ or } DNE, & |r|\geq 1 \end{cases}$$

<u>EX</u>: $1+\frac{1}{2}+\frac{1}{2^2}+\dots = \frac{1}{1-r} = \frac{1}{1-\frac{1}{2}}$

$r=\frac{1}{2}$ $\qquad = \frac{1}{\frac{1}{2}} = 2$

$$\sum_{n=1}^{\infty} 2^{-n} = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\dots$$

$$= \frac{1}{2}\left(1+\frac{1}{2}+\frac{1}{4}+\dots\right) = \frac{1}{2}\cdot 2 = 1$$

FUN EXAMPLE:

$$a_n = \frac{1}{n(n+1)}$$

$$\sum_{n=1}^{\infty} a_n = \cancel{?}\ 1$$

$a_1 = \frac{1}{2}$, $a_2 = \frac{1}{2\cdot 3} = \frac{1}{6}$, $a_3 = \frac{1}{3\cdot 4} = \frac{1}{12}$

$$\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\dots$$

$$\int \frac{1}{x(x+1)}dx = \int \frac{1}{x} - \frac{1}{x+1}$$

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$a_1 = \frac{1}{2} \quad a_2 = \frac{1}{2 \cdot 3} = \frac{1}{2} - \frac{1}{3}$$

$$a_3 = \frac{1}{3 \cdot 4} = \frac{1}{3} - \frac{1}{4}$$

$$a_4 = \frac{1}{4 \cdot 5} = \frac{1}{4} - \frac{1}{5}$$

$$s_1 = a_1 = \frac{1}{2}$$

$$s_2 = a_1 + a_2 = \frac{1}{2} + \frac{1}{2} - \frac{1}{3} = 1 - \frac{1}{3}$$

$$s_3 = s_2 + a_3 = 1 - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \frac{1}{4} = 1 - \frac{1}{4}$$

$$s_4 = s_3 + a_4 = 1 - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \frac{1}{5} = 1 - \frac{1}{5}$$

$$\vdots$$

$$s_n = 1 - \frac{1}{n+1} \quad \lim_{n \to \infty} s_n = 1 - 0 = 1$$