LAST TIME : INTEGRAL CONV. TEST

$a_n = f(n)$

$\sum_{n=1}^{\infty} a_n$ CONVERGES IF AND ONLY IF

$\int_1^{\infty} f(x)\,dx$ CONVERGES

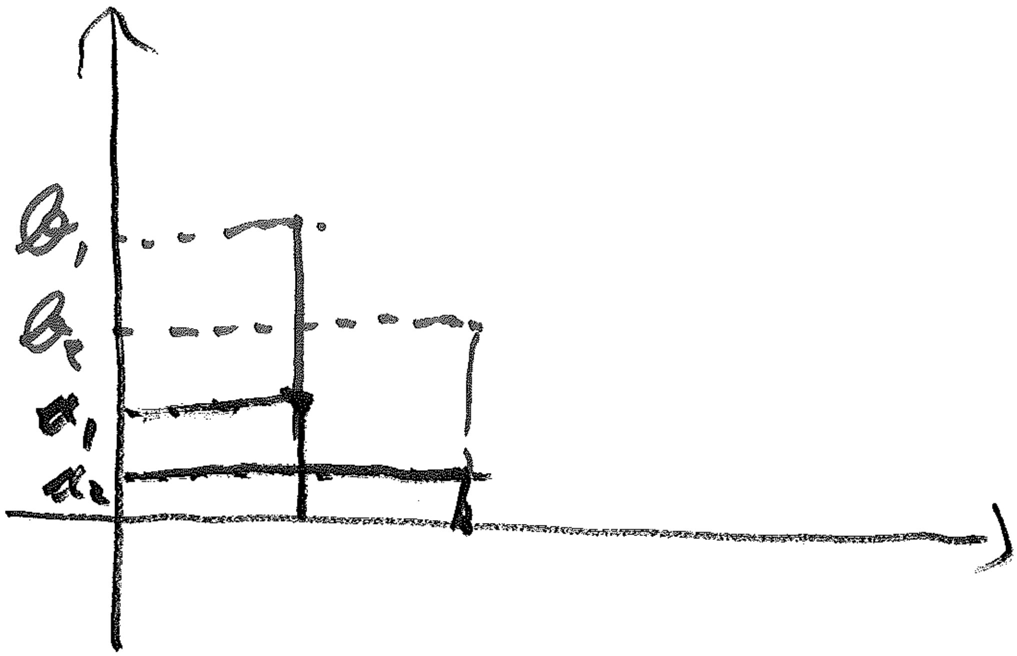
OBS 1: $\sum_{n=5}^{\infty} a_n$ VS. $\int_5^{\infty} f(x)\,dx$

$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ $\qquad$ $\int_2^{\infty} \frac{1}{x \ln x}\,dx$

OBS 2: $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{N} a_n + R_N$

$R_N = a_{n+1} + a_{n+2} + \ldots$

COMPARISON TESTS



Compare $\sum a_n$ with $\sum b_n$

$\{a_n\}, \{b_n\}$ positive, $a_n \leq b_n$

i) If $\{b_n\}$ converges, then $\{a_n\}$ converges

ii) If $\{a_n\}$ diverges, then $\{b_n\}$ diverges

<u>EX1</u>: $a_n = \dfrac{2^n}{3^n+1}$, $\quad b_n = \dfrac{2^n}{3^n}$

$\sum_{n=1}^{\infty} a_n$ CONV. $\qquad a_n \leq b_n$? ✓

$\sum_{n=1}^{\infty} b_n$ CONV. $\qquad \dfrac{2^n}{3^n+1} \leq \dfrac{2^n}{3^n} \qquad 3^n \leq 3^n+1 \qquad 0 \leq 1$ ✓

EX 2: $a_n = \frac{2+e^{-n}}{n}$, $b_n = \frac{2}{n}$

$\sum_{n=1}^{\infty} a_n$ DIVERGES $a_n \geq b_n$

$\sum_{n=1}^{\infty} b_n = 2\sum_{n=1}^{\infty} \frac{1}{n}$ DIVERGES

WHAT IF THE INEQUALITIES ARE HARD, OR GO THE WRONG WAY?

EX 3: $\sum_{n=2}^{\infty} \frac{1}{n^2-1} \approx \sum_{n=2}^{\infty} \frac{1}{n^2}$ CONV.

$$\frac{1}{n^2-1} \overset{?}{\leq} \frac{1}{n^2} \quad n^2 \overset{?}{\leq} n^2-1 \quad \times$$

LIMIT COMPARISON TEST

$\{a_n\}$, $\{b_n\}$ positive

ASSUME THAT $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, $\boxed{0 < L < +\infty}$

THEN $\{a_n\}$ converges if $\{b_n\}$ converges

$\{a_n\}$ diverges if $\{b_n\}$ diverges

EX 3 (redo)

$$\sum_{n=2}^{\infty} \frac{1}{\underbrace{n^2-1}_{a_n}} \quad \text{CONV.} \qquad \sum_{n=2}^{\infty} \frac{1}{\underbrace{n^2}_{b_n}} \quad \text{CONV.}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{1}{n^2-1}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2-1}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^2}}{\cancel{n^2}(1-\frac{1}{n^2})} = \lim_{n\to\infty} \frac{1}{1-\frac{1}{n^2}} = 1$$

EX 4: $a_n = \dfrac{n^3+3n+1}{n^4+5} \approx \dfrac{n^3}{n^4} = \dfrac{1}{n} = b_n$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n^3+3n+1}{n^4+5}}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{n(n^3+3n+1)}{n^4+5} = \lim_{n\to\infty} \frac{\cancel{n^4}(1+\frac{3}{n^2}+\frac{1}{n^3})}{\cancel{n^4}(1+\frac{5}{n^4})}$$

$$= \lim_{n\to\infty} \frac{1+\frac{3}{n^2}+\frac{1}{n^3}}{1+\frac{5}{n^4}} = \frac{1}{1} = 1$$

$\sum b_n$ diverges $\quad \sum a_n$ diverges

WHAT IF $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$?

THEN, IF $\sum b_n$ CONVERGES, THEN

$\sum a_n$ CONVERGES

IF $\sum b_n$ DIVERGES ???

EX: $\sum_{n=1}^{\infty} \underbrace{\frac{e^{-n}}{n^2}}_{a_n}$ CONV. $\sum_{n=1}^{\infty} \underbrace{\frac{1}{n^2}}_{b_n}$ CONV.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} e^{-n} = 0$$

EX: $\sum_{n=1}^{\infty} \frac{e^{-n}}{n}$?? $\sum_{n=1}^{\infty} \frac{1}{n}$ DIVERGES

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} e^{-n} = 0$$

$\sum \frac{e^{-n}}{n}$ CONV. $\sum \frac{1}{n^2}$ CONV.