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BETTER?

TRIGONOMETRIC INTEGRALS

$\int \sin x\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\ln|\cos x| + C$

$u = \cos x$

$= \ln|\sec x| + C$

$\int \sec^2 x\,dx = \tan x$

$\int \sec x\,dx = \ln|\sec x + \tan x| + C$

(see book)

TRIG. IDENTITIES

$\sin^2 x + \cos^2 x = 1$

$\cos 2x = \cos^2 x - \sin^2 x$

SUBTRACT

$\sin^2 x = \frac{1-\cos 2x}{2}$

$\sin 2x = 2\sin x \cos x$

$\tan^2 x = -1 + \sec^2 x$ (try to derive)

$$\int \sin^3 x\,dx = \int \sin^2 x \cdot \sin x\,dx$$

$$= \int (1-\cos^2 x)\sin x\,dx$$

$$u = \cos x$$
$$du = (-\sin x)\,dx$$

$$\int (1-u^2)\cdot(-du)$$

$$= \int (u^2-1)\,du = \frac{u^3}{3} - u + C$$

$$= \frac{1}{3}\cos^3 x - \cos x + C$$

$$\int \sin^3 x \cos x\,dx = \int u^3\,du = \frac{u^4}{4} + C$$

$$u = \sin x \qquad = \frac{\sin^4 x}{4} + C$$
$$du = \cos x\,dx$$

$$\int F(\sin x)\cos x\,dx = \int F(u)\,du$$

$$u = \sin x$$
$$du = \cos x\,dx$$

$$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx$$

$$= \int \frac{1}{2} - \frac{1}{2}\cos 2x\,dx$$

$$= \int \frac{1}{2}\,dx - \int \frac{1}{2}\cos 2x\,dx$$

$$\boxed{\begin{aligned} &\int \frac{1}{2}\cos 2x\,dx = \int \frac{1}{2}\cos u \cdot \frac{1}{2}\,du = \frac{1}{4}\int \cos u\,du \\ &u = 2x,\ du = 2dx \qquad = \frac{1}{4}\sin 2x \end{aligned}}$$

$$= \frac{x}{2} - \frac{1}{4}\sin 2x + C$$

Find formula for $\cos^2 x$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$\int \sin x \boxed{\cos x\,dx}$ — double angle, IBP, substitution

$\sin x = u$

$du = (\sin x)'dx = \cos x\,dx$

$$= \int u\,du = \frac{u^2}{2} + C = \frac{1}{2}\sin^2 x + C$$

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$$\int F(\cos x)\sin x\,dx = -\int F(u)\,du$$

$\cos x = u$

$\sin x\,dx = -du$

$$\int \sin^3 x \cdot \cos^8 x\,dx$$

$$= \int \sin^2 x \cdot \cos^8 x \boxed{\sin x\,dx}$$

$$= \int (1-\cos^2 x)\cos^8 x \sin x\,dx = -\int (1-u^2)u^8\,du$$

$u = \cos x$

$$\int \sin^m x \cdot \cos^n x\,dx$$

i) if m is odd, write as

$$\int \sin^{m-1} x \cos^n x \boxed{\sin x\,dx}$$

$$\sin^{m-1} x = (\sin^2 x)^{\frac{m-1}{2}} = (1-\cos^2 x)^{\frac{m-1}{2}}$$

$u = \cos x$

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ii) if n is odd, write as

$$\int \sin^m x \cdot \cos^{n-1} x \cdot \cos x \, dx$$

$$u = \sin x$$

iii) If m, n are both even, double angle formula (or be clever)

$$\int \sin^2 x \cos^2 x \, dx = \int (\sin x \cos x)^2 \, dx$$

$$= \int \left(\frac{1}{2} \sin 2x\right)^2 dx = \ldots$$

$$\int \tan^3 x \, dx = \int \tan x \cdot \tan^2 x \, dx$$

$$= \int \tan x \cdot (-1 + \sec^2 x) \, dx$$

$$= -\int \tan x \, dx + \int \tan x \cdot \sec^2 x \, dx$$

$$= -\ln|\sec x| + \frac{\tan^2 x}{2} + C \qquad u = \tan x$$