Title : An Inverse Eigenvalue Problem for the Schrödinger Equation on the Unit Ball of $\mathbb{R}^3$.

Abstract

The inverse eigenvalue problem for a given operator is to determine the coefficients by using knowledge of its eigenfunctions and eigenvalues. These are determined by the behavior of the solutions on the domain boundaries. In our problem, the Schrödinger operator acting on functions defined on the unit ball of $\mathbb{R}^3$ has a radial potential taken from $L^2_{\mathbb{R}}[0,1]$. Hence the set of the eigenvalues of this problem is the union of the eigenvalues of infinitely many Sturm-Liouville operators on $[0,1]$ with the Dirichlet boundary conditions. Each Sturm-Liouville operator corresponds to an angular momentum $l = 0, 1, 2....$. In this research we focus on the uniqueness property. This is, if two potentials $p, q \in L^2_{\mathbb{R}}[0,1]$ have the same set of eigenvalues then $p = q$. An early result of Pöschel and Trubowitz is that the uniqueness of the potential holds when the potentials are restricted to the subspace of the even functions of $L^2_{\mathbb{R}}[0,1]$ in the $l = 0$ case. Similarly when $l = 0$, by using their method we proved that two potentials $p, q \in L^2_{\mathbb{R}}[0,1]$ are equal if their even extension on $[-1,1]$ have the same eigenvalues. Also we expect to prove the uniqueness if p and q have the same eigenvalues for finitely many l. For this idea we handle the problem by focusing on some geometric properties of the isospectral sets and trying to use these properties to prove the uniqueness of the radial potential by using finitely many of the angular momentum.