On a Theorem of Wolff Revisited

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Abstract : Fatou's theorem for harmonic functions states that if u is a positive harmonic function in the unit ball or upper half space in $\mathbb{R}^n$ then u has finite radial limits almost everywhere. Tom Wolff proved in 1983 that this theorem fails for solutions to $\nabla(\cdot|\nabla u|^{p-2}\nabla u) = 0$ in the upper half space of $\mathbb{R}^2$ when $p > 2$. A few years later I used a ' conjugate function argument ' to show his theorem generalizes to $1 < p < \infty$. In our paper (to appear Journal De Analysis) with the same name as the above title, we extend Wolff's theorem for $1 < p < \infty$ to the unit disk in $\mathbb{R}^2$.

In this talk we first give some preliminary material on harmonic and p harmonic functions. After that we discuss some of the brilliant ideas Wolff used in his construction. Finally we outline our proof. Time permitting we will also discuss two possible definitions of p harmonic measure and discuss our work in the above paper concerning generalizations of relatively recent work of Llorente, Manfredi, and Wu (on one definition of p harmonic measure in the upper half space) to the unit disk in $\mathbb{R}^2$. .