SPEAKER:

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TITLE:

Compactness and stable regularity in multi-scale homogenization

ABSTRACT:

In this talk, I will talk about the multiscale elliptic equations in the form of $-\mathrm{div}\big(A_\varepsilon \nabla u_\varepsilon\big) = 0$, where $A_\varepsilon(x) = A(x, x/\varepsilon_1, \cdots, x/\varepsilon_n)$ is an n-scale oscillating periodic coefficient matrix, and $(\varepsilon_i)_{1\le i\le n}$ are scale parameters. We show that the C^α-Hölder continuity with any $\alpha \in (0,1)$ for the weak solutions is stable, namely, the constant in the estimate is uniform for arbitrary $(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) \in (0,1]^n$ and particularly is independent of the ratios between ε_i's. The proof uses an upgraded method of compactness, involving a scale-reduction theorem by H-convergence. The Lipschitz estimate for arbitrary $(\varepsilon_i)_{1\le i\le n}$ still remains open. However, for special laminate structures, i.e., $A_\varepsilon(x) = A(x, x_1/\varepsilon_1, \cdots, x_d/\varepsilon)$, we show that the Lipschitz estimate is stable for arbitrary $(\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n) \in (0,1]^n$. This is proved by a technique of reperiodization. This is joint work with Weisheng Niu.