

# The Bernstein Problem in Two Dimensions

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I will outline a complete solution to the weighted approximation problem for polynomials on an arbitrary bounded simply connected domain  $\Omega$  in the complex plane. In that setting the problem was first studied extensively by Keldysh prior to 1941 in the context of  $L^2$ -approximation, and more than four decades later by Beurling where, in the latter instance, the emphasis was on uniform approximation. Here, Beurling obtains the sharper result with respect to the weight  $w$ , but at the expense of limiting the type of region to which his argument applies. Ironically, however, the two problems turned out to be essentially equivalent, but neither Beurling nor Keldysh obtained what might be considered a definitive solution. My presentation will focus on the  $L^2$ -case, where the theory of Sobolev spaces and its associated potential theory is available. It is a simple matter to pass from there to a solution in the case of uniform approximation.