Pointwise wave decay in the asymptotically flat stationary setting

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Abstract

The current work considers solutions to the wave equation on asymptotically flat, stationary spacetimes in (1+3) dimensions. We investigate the relationship between the rate at which the geometry tends to flat and the pointwise decay rate of waves. Sharp Huygens Principle tells us that waves decay infinitely fast in the flat setting. In Tataru 2013 a t^{-3} decay rate was found when the spacetime tends toward flat at a rate of $|x|^{-1}$. Tatarus result proved a long-standing conjecture about wave decay on the Schwarzschild spacetime, which describes the geometry due to a single, stationary black hole. In the current work we obtain pointwise decay for geometries in between the known cases where the geometry is curved but tends toward flat at a rate faster than $|x|^{-1}$. We include a weak local energy decay assumption on the evolution of the equation. Geometrically this corresponds to assuming there are no stable trapped sets on the manifold (i.e. there are no geodesics which stay within a compact region). The weak local energy decay estimate is also deeply connected to the existence of the resolvent at zero frequency. We take advantage of this connection to obtain the final decay rate.