Multi-solitary waves of the Benjamin–Ono equation on the line

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Abstract This presentation is based on the work Sun [2]. The Benjamin–Ono (BO) equation, which describes internal long waves of deep stratified fluids, has multi-soliton solutions. I shall prove the invariance of the multi-soliton manifold $\mathcal{U}_N$, given by

$$\mathcal{U}_N := \{u \in L^2(\mathbb{R},\mathbb{R}) : u(x) = \sum_{j=1}^{N} \frac{2\eta_j}{(x-x_j)^2+\eta_j^2}, \quad \eta_j > 0, \quad x_j \in \mathbb{R}, \quad \forall 1 \le j \le N\}, \quad \forall N \ge 1,$$

under the BO flow and construct global (generalized) action–angle coordinates of the BO equation on $\mathcal{U}_N$ in order to solve this equation by quadrature for any initial datum $u_0 \in \mathcal{U}_N$. The complete integrability of the BO equation on every $\mathcal{U}_N$ constitutes a first step towards the soliton resolution conjecture of the BO equation on the line. The construction of such coordinates relies on the Lax pair structure of the BO equation, the inverse spectral transform and the use of a generating functional, which encodes the entire BO hierarchy. The inverse spectral formula of an N-soliton $u \in \mathcal{U}_N$ provides a spectral connection between the Lax operator and the infinitesimal generator of the shift semigroup acting on some Hardy spaces. Furthermore, $\mathcal{U}_N$ can be interpreted as the universal covering of the manifold of N-gap potentials $U_N^{\mathbb{T}}$ for the space-periodic BO equation as described by Gérard–Kappeler [1].

Keywords Benjamin–Ono equation, multi-solitons, global well-posedness, generalized action–angle coordinates, Lax pair, shift semigroup on Hardy space, universal covering manifold

References

[1] Gérard, P., Kappeler, T. *On the integrability of the Benjamin–Ono equation on the torus*, Commun. Pure Appl. Math. **74** (2021), no.8, 1685-1747, https://doi.org/10.1002/cpa.21896, 2021.

[2] Sun, R. *Complete integrability of the Benjamin–Ono equation on the multi-soliton manifolds*, Commun. Math. Phys. **383**, 1051–1092 (2021). https://doi.org/10.1007/s00220-021-03996-1

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